THE SUMMER MEETING IN CAMBRIDGE

The forty-second Summer Meeting of the Society and the nineteenth Colloquium were held at Harvard University, Cambridge, Massachusetts, in connection with the Harvard Tercentenary Conference of Arts and Sciences, from Monday, August 31 to Saturday, September 5. This was the fourth meeting of the Society to be held in Cambridge. It was the second Summer Meeting held there, the earlier one having been in September, 1916. The other two meetings were held in conjunction with the American Association for the Advancement of Science in December, 1922, and in December, 1933, the Massachusetts Institute of Technology then participating as one of the hosts.

Harvard University, celebrating its Tercentenary, was a gracious host, and hospitality was unbounded. All lectures given during the week under the auspices of the Tercentenary Committee were open to the mathematicians. More than half of those attending were housed without charge in the Yard. There were a large number of excursions and social diversions. All the facilities of the University were at the disposal of the visitors, who were unanimous in praise of the arrangements made for their comfort and convenience. At the Saturday morning session the following resolution was adopted by standing vote:

RESOLVED: that we, the members of the American Mathematical Society and the Mathematical Association of America, here assembled for the largest meeting in the history of our organizations, express our profound appreciation to the President and Fellows of Harvard College and to the Tercentenary Committee for sharing with us the distinguished guests of the Tercentenary Conference and for the most generous provision which they have made for us in the buildings of Harvard University; to Mr. Greene, Mr. Land, and Mr. McNeil of the Tercentenary Committee for the painstaking attention which they have given to our comfort and convenience; to the Division of Mathematics and the Committee on Arrangements, including especially Dean and Mrs. Birkhoff and Professor and Mrs. Graustein, for the extraordinary care with which they have provided for our sessions and our entertainment and for the exhibition of notable mathematical books and of the work of the Division; and to the ladies of the Division of Mathematics, to President McAfee and Professor Young of Wellesley College, and to Professor and Mrs. Shapley for their gracious hospitality.

The Mathematical Association of America, the Institute of Mathematical Statistics, the Association for Symbolic Logic, and the American Astronomical Society held sessions during the same week. Among the more than one thousand persons attending the meetings arranged by the Society, approximately eight hundred registered, of whom 443 are members of the Society. This constitutes a record attendance, an excess of fifty per cent more than at the largest meeting hitherto. Among the members of the Society present, there were forty-one states represented, and five provinces of Canada, besides Europe and Asia. The largest delegation (76) was from Massachusetts, followed by New York (72), Pennsylvania (34), New Jersey (31), Ohio (23), Illinois (21), Rhode Island (18), Michigan (17), Connecticut (13).

The following members of the Society were in attendance:

C. R. Adams, R. B. Adams, V. W. Adkisson, C. F. Adler, H. L. Agard, R. P. Agnew, E. B. Allen, C. B. Allendoerfer, R. C. Archibald, R. G. Archibald, L. A. Aroian, W. L. Ayres, R. W. Babcock, C. L. Bacon, H. M. Bacon, Reinhold Baer, D. H. Ballou, G. M. Bareis, J. L. Barnes, I. A. Barnett, J. H. Bartlett, Ralph Beatley, Samuel Beatty, M. F. Becker, E. R. Beckwith, R. D. Beetle, A. A. Bennett, D. L. Bernstein, Felix Bernstein, Garrett Birkhoff, G. D. Birkhoff, Gertrude Blanch, H. F. Blichfeldt, G. A. Bliss, R. P. Boas, H. F. Bohnenblust, J. W. Bower, M. G. Boyce, C. B. Boyer, J. W. Bradshaw, Richard Brauer, R. W. Brink, H. W. Brinkmann, Samuel Bronstein, B. H. Brown, B. L. Brown, E. P. Brown, E. W. Brown, O. E. Brown, T. H. Brown, R. E. Bruce, G. S. Bruton, N. R. Bryan, H. E. Buchanan, W. M. Bullitt, L. H. Bunyan, R. S. Burington, F. W. Burton, Jewell H. Bushey, J. H. Bushey, W. H. Bussey, S. S. Cairns, W. D. Cairns, R. H. Cameron, B. H. Camp, A. D. Campbell, G. A. Campbell, M. E. Carlen, F. E. Carr, E. J. Cartan, W. B. Carver, E. W. Chittenden, Alonzo Church, R. N. Cobb, A. B. Coble, Abraham Cohen, L. W. Cohen, Nancy Cole, J. B. Coleman, Esther Comegys, E. G. H. Comfort, H. R. Cooley, J. L. Coolidge, G. A. Coon, T. F. Cope, A. H. Copeland, Byron Cosby, Richard Courant, Carleton Craig, C. C. Craig, A. R. Crathorne, W. L. Crum, C. H. Currier, H. B. Curry, D. R. Curtiss, J. H. Curtiss, E. H. Cutler, D. R. Davis, W. M. Davis, F. F. Decker, W. W. Denton, L. E. Dickson, Vincent Dillon, C. E. Dimick, L. L. Dines, E. C. Doak, J. L. Doob, R. D. Douglass, T. L. Downs, Arnold Dresden, D. M. Dribin, Nelson Dunford, J. C. Durand, W. L. Duren, W. H. Durfee, J. M. Earl, W. W. Elliott, G. C. Evans, G. W. Evans, H. S. Everett, Fay Farnum, Peter Field, T. S. Fiske, W. B. Fite, D. A. Flanders, M. M. Flood, L. R. Ford, Tomlinson Fort, I. G. Foster, R. M. Foster, J. S. Frame, C. W. Franklin, Philip Franklin, A. H. Frink, Orrin Frink, T. C. Fry, D. L. Fuller, A. S. Galbraith, A. S. Gale, C. A. Garabedian, H. L. Garabedian, H. J. Gay, J. J. Gergen, B. C. Getchell, K. S. Ghent, J. L. Gibson, Lachlan Gilchrist, R. E. Gilman, Michael Goldberg, S. H. Gould, A. A. Grant, M. C. Graustein, W. C. Graustein, L. M. Graves, L. W. Griffiths, V. G. Grove, H. V. Gummere, Marshall Hall,

Harris Hancock, G. H. Hardy, J. G. Hardy, R. A. Harrison, G. G. Harvey, T. W. Hatcher, Alan Hazeltine, G. A. Hedlund, E. R. Hedrick, C. B. Hennel, Max Herzberger, D. M. Hickey, H. C. Hicks, A. O. Hickson, E. H. C. Hildebrandt, T. H. Hildebrandt, R. C. Hildner, Einar Hille, F. H. Hodge, T. F. Holgate, T. R. Hollcroft, Eberhard Hopf, Charles Hopkins, L. A. Hopkins, E. M. Hull, E. V. Huntington, C. A. Hutchinson, M. H. Ingraham, Dunham Jackson, R. F. Jackson, R. L. Jeffery, E. D. Jenkins, Evan Johnson, M. M. Johnson, R. A. Johnson, R. P. Johnson, F. E. Johnston, B. W. Jones, H. E. Jordan, M. L. Kales, L. C. Karpinski, Edward Kasner, L. O. Kattsoff, D. E. Kearney, A. J. Kempner, L. S. Kennison, G. S. Ketchum, P. W. Ketchum, S. H. Kimball, H. R. Kingston, A. D. Kinsman, S. C. Kleene, J. R. Kline, B. O. Koopman, R. L. Korgen, A. C. Ladner, A. W. Landers, M. K. Landers, A. E. Landry, Otto Laporte, E. Kramer Lassar, V. V. Latshaw, V. S. Lawrence, D. H. Leavens, Solomon Lefschetz, A. M. M. Lehr, D. D. Leib, Walter Leighton, F. E. LeStourgeon, Tullio Levi-Civita, Harry Levy, F. P. Lewis, H. G. Lieber, L. R. Lieber, P. H. Linehan, G. H. Ling, Marie Litzinger, W. R. Longley, L. L. Lowenstein, E. D. McCarthy, R. B. McClenon, Dorothy McCoy, N. H. McCoy, E. A. McDougle, W. H. McEwen, J. C. C. McKinsey, Saunders MacLane, H. M. MacNeille, H. F. MacNeish, H. P. Manning, R. J. Marcou, Morris Marden, R. S. Martin, W. T. Martin, R. G. Mason, T. E. Mason, S. P. Mead, A. E. Meder, H. A. Merrill, G. M. Merriman, A. D. Michal, F. F. Middleswart, D. C. Miller, W. I. Miller, E. B. Mode, Deane Montgomery, C. N. Moore, E. M. Morenus, R. K. Morley, Max Morris, Richard Morris, D. S. Morse, Z. I. Mosesson, T. A. Mossman, E. J. Moulton, F. D. Murnaghan, S. B. Myers, J. H. Neelley, C. A. Nelson, John von Neumann, A. V. Newton, R. E. O'Connor, W. F. Osgood, F. W. Owens, H. B. Owens, J. C. Oxtoby, Gordon Pall, W. O. Pennell, W. F. Penney, F. W. Perkins, R. M. Peters, H. B. Phillips, A. E. Pitcher, Hillel Poritsky, E. C. Pousland, G. B. Price, H. A. Rademacher, Tibor Radó, G. Y. Rainich, W. C. Randels, J. F. Randolph, W. R. Ransom, H. W. Reddick, C. J. Rees, W. T. Reid, C. N. Reynolds, Harris Rice, A. V. Richardson, R. G. D. Richardson, C. A. Richmond, D. E. Richmond, H. L. Rietz, David Rines, John Riordan, J. F. Ritt, H. P. Robertson, R. M. Robinson, Robin Robinson, S. L. Robinson, W. B. Robinson, W. H. Roever, R. E. Root, J. B. Rosenbach, J. B. Rosser, N. N. Royall, H. A. Ruger, E. C. Rusk, W. J. Rusk, H. G. Russell, W. M. Rust, George Rutledge, N. E. Rutt, O. K. Sagen, George Sauté, A. C. Schaeffer, S. A. Schelkunoff, I. J. Schoenberg, H. E. Schoonmaker, Wladimir Seidel, I. M. Sheffer, J. A. Shohat, L. L. Silverman, W. G. Simon, T. M. Simpson, E. R. Sleight, M. M. Slotnick, C. H. Smiley, H. L. Smith, T. L. Smith, W. M. Smith, E. S. Sokolnikoff, I. S. Sokolnikoff, Joseph Spear, V. E. Spencer, A. A. Stafford, G. W. Starcher, M. E. Stark, E. P. Starke, R. C. Stauffer, J. Y. Stephens, R. C. Stephens, R. W. Stokes, M. H. Stone, R. B. Stone, W. T. Stratton, R. E. Street, D. J. Struik, R. R. Struik, M. C. Suffa, M. M. Sullivan, Elijah Swift, Gabriel Szegö, J. D. Tamarkin, A. E. Taylor, J. H. Taylor, J. S. Taylor, M. E. Taylor, W. C. Taylor, J. M. Thomas, T. Y. Thomas, Helen Thompson, H. S. Thurston, C. C. Torrance, M. M. Torrey, J. I. Tracey, J. C. Trainor, W. J. Trjitzinsky, A. W. Tucker, C. B. Tucker, J. W. Tukey, H. L. Turrittin, H. W. Tyler, F. E. Ulrich, J. L. Vander-

slice, J. H. Van Vleck, T. Vijayaraghavan, G. P. Wadsworth, G. W. Walker, R. J. Walker, H. S. Wall, J. L. Walsh, R. M. Walter, T. O. Walton, L. E. Ward, Morgan Ward, J. F. Wardwell, W. G. Warnock, S. E. Warschawski, C. W. Watkeys, Warren Weaver, D. W. Weeks, F. M. Weida, E. T. Welmers, Albert Wertheimer, A. H. Wheeler, O. L. Wheeler, H. S. White, M. B. White, E. A. Whitman, Hassler Whitney, J. K. Whittemore, G. T. Whyburn, D. V. Widder, Norbert Wiener, E. P. Wiggin, E. P. Wigner, C. E. Wilder, R. L. Wilder, S. S. Wilks, Ruby Willis, E. B. Wilson, E. W. Wilson, R. M. Winger, Clement Winston, E. E. Witmer, L. A. Wolf, M. C. Wolf, F. S. Woods, F. M. Wright, B. F. Yanney, C. H. Yeaton, M. M. Young, E. I. Yowell, Oscar Zariski, S. D. Zeldin.

One of the chief features of the meeting was the Series of Harvard Tercentenary Conference Lectures delivered by eminent mathematicians, both foreign and American, whom Harvard University had invited as speakers. These were delivered in the New Lecture Hall and were all broadcast on both long and short wave. The following is a list of these addresses delivered by the courtesy of Harvard University before the Society:

I. Monday afternoon. *Uncertain inference.* By Ronald Aylmer Fisher, Sc.D., Professor of Eugenics, University of London.

II. Monday evening. *The Indian mathematician, Ramanujan.* By Godfrey Harold Hardy, D.Sc., LL.D., D.Phil., Professor of Mathematics, University of Cambridge.

III. Tuesday morning. *Truth in mathematics and logic.* By Rudolf Carnap, D.Phil., Professor of Philosophy, Deutsche Universität, Prague.

IV. Tuesday afternoon. *L'extension du calcul tensoriel aux géométries non-affines.* By Élie Joseph Cartan, D.Sc., Professor of Mathematics, University of Paris.

V. Wednesday morning. *Waring's problem and its generalizations.* By Leonard Eugene Dickson, Ph.D., Professor of Mathematics, University of Chicago.

VI. Thursday morning. *The mathematical work of Ramanujan.* By Godfrey Harold Hardy, D.Sc., LL.D., D.Phil., Professor of Mathematics, University of Cambridge.

VII. Friday morning. *The relativistic problem of several bodies.* By Tullio Levi-Civita, Dr.Math., D.Sc., Professor of Rational Mechanics, University of Rome.

VIII. Friday afternoon. *Astronomical consequences of the relativistic two-body problem.* By Tullio Levi-Civita, Dr.Math., D.Sc., Professor of Rational Mechanics, University of Rome.

IX. Friday afternoon. *The cosmical constant and the recession of the nebulae.* By Sir Arthur Stanley Eddington, Sc.D., LL.D., Professor of Astronomy, University of Cambridge.

Lecture I was delivered before a joint session with the Mathematical Association of America and the Institute of Mathematical Statistics at which Professor H. L. Rietz presided; II before a joint session with the M.A.A., Professor J. L. Coolidge presiding; III before a joint session with the Association for Symbolic Logic, a sectional meeting which was arranged by Professor H. B. Curry and presided over by Professor A. A. Bennett. C. J. Ducasse, Professor of Philosophy at Brown University, introduced Professor Carnap. There were 350 persons at this session and 50 attended a special luncheon immediately following at the Faculty Club at which Professor A. N. Whitehead gave an hour address, Professor E. V. Huntington presiding. Professor H. S. White presided at Lecture IV, Professor A. B. Coble at V, Professor T. S. Fiske at VI, Professor E. W. Brown at VII. Lectures VIII and IX were given before a joint session with the American Astronomical Society, Dean G. D. Birkhoff presiding; and these were followed by discussion on the part of Professor Arthur Haas of the University of Vienna and Professor H. P. Robertson of Princeton University. All of these addresses will be published in the journals of the Society or elsewhere.

The Colloquium Lectures entitled *Topics in general analysis* were delivered by Professor E. W. Chittenden of the University of Iowa on Tuesday, Wednesday, and Friday mornings and on Thursday afternoon in Emerson Hall. The attendance of 220 establishes a new record. Each lecture was approximately one hour in length. The series will appear in book form. The presiding officers at these lectures were President Lefschetz, Vice-President Norbert Wiener, and Professors E. R. Hedrick and F. S. Woods.

By invitation of the Committee on Program, Professor G. C. Evans of the University of California gave on Saturday morning an address entitled *Methods of modern analysis in potential theory.* This address will appear in an early issue of this Bulletin.

Besides the meetings mentioned above nine sessions of the Society for the presentation of regular papers were held in Emerson Hall on Tuesday, Thursday, and Saturday mornings, and on Tuesday and Thursday afternoons, the presiding officers be-

ing Professors G. A. Bliss, E. V. Huntington, H. B. Phillips, M. H. Ingraham, J. L. Walsh, H. F. Blichfeldt, W. C. Graustein, W. F. Osgood, and J. D. Tamarkin.

The Mathematical Association held sessions on Monday morning and afternoon.

The Tercentenary Conference arranged several evening lectures to which mathematicians were invited. Besides that of Professor Hardy given on Monday evening at the joint session of the Society and the Association, there was on Wednesday evening an address by Professor Étienne Gilson of the Collège de France on *Medieval universalism and its present value*, on Thursday evening one by Professor Edward Joseph Dent of the University of Cambridge on *The historical approach to music*, and on Friday evening one by Professor John Dewey of Columbia University on *Authority and resistance to social change*.

On Thursday evening an informal dinner for the members of the Society and the Association and their guests was held at the Copley-Plaza; 520 people attended this dinner. Professor J. L. Coolidge acted as toastmaster and there were addresses by President J. B. Conant and Dean G. D. Birkhoff of Harvard University, by Professor G. H. Hardy on behalf of the guests, by President Lefschetz of the Society, and by Professor G. A. Bliss.

Professor and Mrs. Harlow Shapley entertained the visiting mathematicians and astronomers at a garden party on Thursday afternoon at the Harvard Observatory residence. Every afternoon tea was served in Phillips Brooks House, the social headquarters for the mathematicians, by the ladies of the Department of Mathematics.

The numerous excursions planned by the Committee on Arrangements added greatly to the pleasure of the meetings and afforded additional opportunities for social intercourse. At the close of the Monday afternoon session there was a short trip in busses around historic Boston, with guides in attendance.

For late Tuesday afternoon two excursions were arranged. One party visited the Massachusetts Institute of Technology to inspect important machines, charts, and the differential analyzer. The second party went to Wellesley College to visit the campus and were received by President M. H. McAfee and Professor M. M. Young; tea was served in Tower Court.

Wednesday afternoon was devoted to an excursion to Lexington and Concord and to the Wayside Inn, of Longfellow fame, at Sudbury. At the Antiquarian Society building in Concord, Emerson's library was seen. Guides pointed out other spots of interest and a return was made through the Fenway.

On Thursday afternoon guides took parties to visit various Harvard buildings, among them the Faculty Room of University Hall, the University Museum including the Botanical Museum with its display of glass flower models, and the Peabody Museum with its exhibition on the culture of human races.

Excursions were arranged for wives and children of members. On Monday there was a trip through the Yard; on Tuesday to the Isabella Stewart Gardner Museum; on Wednesday to the Tercentenary Exhibition of Furniture and the Decorative Arts and to the Fogg Art Museum; on Thursday to the Boston Museum of Fine Arts; and on Friday to the Undergraduate Houses, the Longfellow House, and Old Cambridge.

The last event of the meetings, in the form of an excursion to the North Shore with a shore dinner at the Adams House in Marblehead, took place late on Saturday morning. On the return trip there was an opportunity, for those who desired, to visit Salem, where guides conducted them to points of interest.

On Thursday afternoon at 1:30 p.m. a panoramic picture of the mathematicians and guests was taken on the steps of Widener Library. This proved to be an interesting feature and approximately 150 copies were sold.

The mathematicians were fortunate in being able to enjoy Harvard's noteworthy exhibits which included mathematical books and a special collection pertaining to the history and scientific activities of the department. Massachusetts Institute of Technology held open house on Tuesday and Thursday for those attending the meetings.

Meals were served in the Harvard Union. Arrangements were made through the Harvard Athletic Association for golf, tennis, and boating and the Harvard Tercentenary Information and Guide Service offered its efficient cooperation to those mathematicians who desired to make individual excursions.

The Council met on Tuesday, September 1, at 8 p.m., in the Library of the Faculty Club. The Board of Trustees held a meeting on Thursday at 12:30 p.m. in the Harvard Union.

The Secretary announced the election of the following persons to membership in the Society:

Mr. George Richmond Carter, Soil Conservation Service, Marion, Illinois;
Miss Ruth M. Culbertson, Highland College, Highland, Kansas;
Dr. Kenneth Smith Ghent, University of Oregon;
Dr. Max Herzberger, Eastman Kodak Research Laboratories, Kodak Park, Rochester, N. Y.;
Dr. Archie Higdon, Boone, Iowa;
Mr. Frederick Gilbert King, Works Progress Administration, New York, N.Y.;
Mr. Fred Kiokemeister, University of Wisconsin;
Mr. Jack Laderman, Bronx, N. Y.;
Mr. Henry Hollis Lansford, Brooklyn College;
Sister Mary Christopher, St. Augustine High School, Kalamazoo, Michigan;
Mr. Francis Fulton Middleswart, Brown University;
Miss Zoé Elizabeth O'Ferrall, Gulf Park College, Gulfport, Mississippi;
Mr. John C. Oxtoby, Harvard University;
Mr. Harwood Rosser, Jr., Jacksonville, Florida;
Dr. Ruth Caroline Stauffer, Harrisburg, Pennsylvania;
Dr. Angus Ellis Taylor, Pasadena, California;
Dr. Stanislaw M. Ulam, Lwow, Poland.

As nominee of the Massachusetts Institute of Technology:

Mr. A. C. Shaeffer, Massachusetts Institute of Technology.

As nominee of Michigan State College:

Mr. Paul L. Dressel, Michigan State College.

The following five institutions were elected to institutional membership in the Society:

Beloit College;
Brigham Young University;
University of Manitoba;
University of Southern California;
Department of Mathematics of Wayne University.

The following appointments by Professor Lefschetz were reported: as an additional member of the Committee on Arrangements for the 1936 Annual Meeting, Professor E. T. Browne; as the Committee on Arrangements for the Gibbs Lecture, Professor B. P. Gill (chairman), Professor W. B. Fite, and Dr. L. A. MacColl; as the reappointed representative of the Society on the Sectional Committee on Standards for Graphic Presentation, Professor W. H. Roever; as the representative of the Society at the inauguration of Grover C. Dillman as President of the Michigan College of Mining and Technology, Houghton,

Michigan, in connection with the celebration of the fiftieth anniversary of the founding of the college, Professor C. C. Spooner; as the Committee to consider recommendations for the use of the Marion Reilly Fund, Professors D. V. Widder (chairman), G. A. Hedlund, R. E. Langer, J. D. Tamarkin, and T. Y. Thomas.

It was announced that Professor Henry Norris Russell will be the 1936 Gibbs Lecturer, the lecture to be delivered at a joint meeting of the Society and the American Institute of Physics on October 31 in New York City.

A report was presented from Dean L. P. Eisenhart, as chairman of the delegates of the American Mathematical Society to the International Congress for Mathematicians at Oslo, stating that the invitation from the Society to hold the next Congress in this country in 1940 had been unanimously accepted. A committee was constituted to map out a general plan for the Congress, the membership being: Dean L. P. Eisenhart (general chairman), Professors G. A. Bliss, E. W. Chittenden, G. C. Evans, E. R. Hedrick, T. H. Hildebrandt, Einar Hille, J. R. Kline, Marston Morse, P. A. Smith, M. H. Stone, J. D. Tamarkin, J. L. Walsh, and Norbert Wiener.

Announcement was made that by the will of Miss Marion Reilly, former dean of Bryn Mawr College, considerable sums of money have been made and will be made available for the encouragement of research in pure mathematics.

The titles and cross references to abstracts of the papers read at the sessions follow below. The papers were read as follows: papers numbered 1 to 10 in the Section for Foundations on Tuesday morning; papers numbered 11 to 17 in the Section for Analysis on Tuesday morning; papers numbered 18 to 23 in the Section for Topology on Tuesday afternoon; papers numbered 24 to 31 in the Section for Analysis on Tuesday afternoon; papers numbered 32 to 38 in the Section for Algebra on Tuesday afternoon; papers numbered 39 to 43 in the Section for Applied Mathematics on Thursday morning; papers numbered 44 to 50 in the Section for Algebra on Thursday morning; papers numbered 51 to 57 in the Section for Geometry on Thursday morning; papers numbered 58 to 65 on Thursday afternoon; and papers numbered 66 to 68 on Saturday morning; besides, papers num-

bered 69 to 118 by title. Mr. Robbins was introduced by Dean G. D. Birkhoff, Professor Staniland by Dean R. G. D. Richardson, Dr. Saks by Professor J. D. Tamarkin, Mr. Paxson by Professor A. D. Michal, and Dr. Brenner by Mr. Garrett Birkhoff. Paper number 2 was presented by Dr. Kleene, number 13 by Mr. Jordan, number 27 by Dr. Schoenberg, and number 45 by Professor Ingraham. Papers whose abstract numbers are followed by *t* were read by title.

1. *Extensions of some theorems of Gödel and Church*, by Dr. J. B. Rosser. (Abstract No. 42-7-299.)

2. *Constructions of formal definitions of functions of ordinal numbers*, by Professor Alonzo Church and Dr. S. C. Kleene. (Abstract No. 42-9-380.)

3. *Postulates for assertion, conjunction, negation, and equality*, by Professor E. V. Huntington. (Abstract No. 42-7-294.)

4. *A formal theory of types*, by Dr. F. B. Fitch. (Abstract No. 42-9-386.)

5. *The algebras of concentric sectors*, by Professor Paul Henle. (Abstract No. 42-9-385-*t*.)

6. *Group theory for truth values*, by Dr. L. O. Kattsoff. (Abstract No. 42-9-384-*t*.)

7. *The calculus of logical implication*, by Dr. A. F. Emch. (Abstract No. 42-9-383-*t*.)

8. *The self-contradictory*, by Professor Paul Weiss. (Abstract No. 42-9-382-*t*.)

9. *Two practical problems requiring techniques from symbolic logic*, by Mr. E. C. Berkeley. (Abstract No. 42-9-381-*t*.)

10. *An extension of the concept called language*, by Mr. Reinhard Korgen. (Abstract No. 42-9-379-*t*.)

11. *Concerning polynomials simultaneously orthogonal on more than one curve*, by Professor G. M. Merriman. (Abstract No. 42-7-279.)

12. *Stochastic processes depending on a continuously varying parameter*, by Dr. J. L. Doob. (Abstract No. 42-7-288.)

13. *On convergence of continued fractions*, by Mr. J. Q. Jordan and Dr. Walter Leighton. (Abstract No. 42-7-296.)

14. *On the convergence of continued fractions*, by Dr. Walter Leighton. (Abstract No. 42-9-339.)

15. *A problem of Zermelo in the calculus of variations*, by Professor W. L. Duren. (Abstract No. 42-7-289.)

16. *On Kakeya's problem* (preliminary report), by Professor Morris Marden. (Abstract No. 42-9-343.)

17. *Stereographic parameters and pseudo-minimal hypersurfaces.* II, by Professor Otto Laporte. (Abstract No. 42-9-338.)

18. *General distance functions and the metrization problem*, by Dr. Aline H. Frink. (Abstract No. 42-7-291.)

19. *Semi-closed sets and collections*, by Professor G. T. Whyburn. (Abstract No. 42-7-307.)

20. *Branched and folded coverings*, by Professor A. W. Tucker. (Abstract No. 42-7-317.)

21. *When can a graph be mapped on a torus?*, by Dr. Saunders MacLane. (Abstract No. 42-9-341.)

22. *Periodic transformations of sets*, by Professor W. L. Ayres. (Abstract No. 42-9-319.)

23. *Concerning separating transformations which are entirely complete*, by Dr. J. F. Wardwell. (Abstract No. 42-9-376.)

24. *Functions representable by two Laplace integrals*, by Professor D. H. Ballou. (Abstract No. 42-9-321.)

25. *An extension of Bernstein's theorem associated with general boundary value problems*, by Professor W. H. McEwen. (Abstract No. 42-9-344.)

26. *A generalization of the Cauchy-Riemann equations*, by Dr. A. E. Taylor. (Abstract No. 42-9-369.)

27. *Fourier integrals and metric geometry*, by Professor John von Neumann and Dr. I. J. Schoenberg. (Abstract No. 42-9-353.)

28. *Non-linear difference equations*, by Professor W. J. Trjitzinsky. (Abstract No. 42-9-371.)

29. *Sufficient conditions by expansion methods for the problem of Bolza in parametric form*, by Dr. W. T. Reid. (Abstract No. 42-9-361.)

30. *On certain infinite systems of linear equations and an expansion problem of Pincherle*, by Dr. P. W. Ketchum. (Abstract No. 42-9-337.)

31. *On a class of recurrent sequences*, by Mr. H. E. Robbins. (Abstract No. 42-9-362.)

32. *On the degrees of the irreducible components of simply transitive permutation groups*, by Dr. J. S. Frame. (Abstract No. 42-9-318.)

33. *On the reduction of positive quaternary quadratic forms*

(preliminary report), by Professor B. W. Jones. (Abstract No. 42-9-336.)

34. *Applications of the rational automorphs of* $x^2+y^2+z^2$ (preliminary report), by Professor Gordon Pall. (Abstract No. 42-9-354.)

35. *Arithmetical functions on rings*, by Professor Morgan Ward. (Abstract No. 42-9-375.)

36. *On algebras which are connected with the simple continuous groups*, by Professor Richard Brauer. (Abstract No. 42-9-325.)

37. *R-matrices and equivalent networks*, by Professor R. S. Burington. (Abstract No. 42-9-327.)

38. *Abelian groups without elements of finite order*, by Dr. Reinhold Baer. (Abstract No. 42-9-320.)

39. *Recurrence relations for the moments of Bernoulli and Poisson frequency distributions*, by Mr. John Riordan. (Abstract No. 42-7-298.)

40. *A general theorem for rotationally symmetrical optical systems*, by Dr. Max Herzberger. (Abstract No. 42-7-293.)

41. *On finger print patterns and their singular points*, by Dr. Hillel Poritsky. (Abstract No. 42-9-357.)

42. *On universally valid stress-strain relations for an elastic solid possessing an energy of deformation*, by Professor F. D. Murnaghan. (Abstract No. 42-9-351.)

43. *A converse of Poisson's theorem in the theory of probability*, by Professor Salomon Bochner. (Abstract No. 42-7-274-*t*.)

44. *Lie groups simply isomorphic with no linear group*, by Mr. Garrett Birkhoff. (Abstract No. 42-7-286.)

45. *Relative linear sets and similarity of matrices whose elements belong to a division algebra*, by Professor M. H. Ingraham and Dr. Margarete C. Wolf. (Abstract No. 42-7-295.)

46. *Extension of a multiplicative system to a Boolean ring*, by Dr. H. M. MacNeille. (Abstract No. 42-7-316.)

47. *On the "singular series" for sums of squares of polynomials*, by Professor Leonard Carlitz. (Abstract No. 42-7-313-*t*.)

48. *Representation by generalized polygonal numbers*, by Professor L. W. Griffiths. (Abstract No. 42-7-314.)

49. *An isomorphism between linear recurring sequences and algebraic rings*, by Dr. Marshall Hall. (Abstract No. 42-7-277.)

50. *On Waring's problem in algebraic fields*, by Professor Hans Rademacher. (Abstract No. 42-9-360.)

51. *Extensions of the four-vertex theorem*, by Professor W. C. Graustein. (Abstract No. 42-7-276.)

52. *A condition in invariant form for a net without detours*, by Professor Robin Robinson. (Abstract No. 42-9-364.)

53. *Non-trochoidal rational curves of order n which admit dihedral groups of order* $2n-4$, by Professor R. M. Winger. (Abstract No. 42-7-309.)

54. *The binet of quadrics in* S_3, by Professor T. R. Hollcroft. (Abstract No. 42-7-315.)

55. *Einstein spaces of class one*, by Mr. C. B. Allendoerfer. (Abstract No. 42-7-310.)

56. *Analytic affine transformations in euclidean space of* $2n$ *dimensions*, by Professor A. E. Staniland. (Abstract No. 42-9-367.)

57. *On product-continuous sets*, by Dr. C. C. Torrance. (Abstract No. 42-9-370.)

58. *The four color problem*, by Professor Philip Franklin. (Abstract No. 42-7-290.)

59. *Locally connected subsets of euclidean n-space*, by Professor R. L. Wilder. (Abstract No. 42-7-308.)

60. *Orthogonal polynomials on a plane curve*, by Professor Dunham Jackson. (Abstract No. 42-9-335.)

61. *Minimal surfaces of general topological structure with any nite number of assigned boundaries*, by Professor Jesse Douglas. (Abstract No. 42-5-239-*t*.)

62. *On convergence factors in restrictedly convergent double series*, by Professor C. N. Moore. (Abstract No. 42-9-349.)

63. *On products of Nörlund transformations*, by Professor L. L. Silverman. (Abstract No. 42-9-366.)

64. *On a theorem of Fejér and Riesz*, by Dr. E. F. Beckenbach. (Abstract No. 42-9-322-*t*.)

65. *Conformal mapping and Plateau's problem*, by Dr. Richard Courant. (Abstract No. 42-9-388.)

66. *A general formula for polyhedra with applications to higher dodecahedra and icosahedra*, by Mr. A. H. Wheeler. (Abstract No. 42-9-377.)

67. *The tensor calculus in a general Riemannian differential geometry*, by Professor A. D. Michal. (Abstract No. 42-9-346.)

68. *The theory of classes: a modification of von Neumann's system*, by Dr. R. M. Robinson. (Abstract No. 42-9-363.)

69. *Sufficient conditions for various degrees of convergence by polynomials*, by Professor J. L. Walsh and Dr. W. E. Sewell. (Abstract No. 42-5-236-*t*.)

70. *A problem in Factorisatio Numerorum*, by Professor Einar Hille. (Abstract No. 42-7-278-*t*.)

71. *The category and Borel class of certain subsets of* L_p, by Mr. J. C. Oxtoby. (Abstract No. 42-7-281-*t*.)

72. *On some functionals*. II, by Dr. Stanislaw Saks. (Abstract No. 42-7-282-*t*.)

73. *Analytic arcs in analytic manifolds*, by Professor Walther Mayer and Professor T. Y. Thomas. (Abstract No. 42-7-297-*t*.)

74. *On simple analytic arcs*, by Professor T. Y. Thomas. (Abstract No. 42-7-301-*t*.)

75. *On the covering of analytic arcs by coordinate neighborhoods*, by Professor T. Y. Thomas. (Abstract No. 42-7-302-*t*.)

76. *Fields of parallel vectors in the large*, by Professor T. Y. Thomas. (Abstract No. 42-7-303-*t*.)

77. *Riemann spaces of class one and their characterization*, by Professor T. Y. Thomas. (Abstract No. 42-7-304-*t*.)

78. *On an expansion of the inverse Laplace transformation and its application*, by Professor R. V. Churchill. (Abstract No. 42-7-287-*t*.)

79. *Expansions involving a system of differential equations in which the coefficients of the parameters change sign*, by Professor C. C. Camp. (Abstract No. 42-7-311-*t*.)

80. *Fuchsian groups and transitive horocycles*, by Professor G. A. Hedlund. (Abstract No. 42-9-332-*t*.)

81. *Two-dimensional manifolds and transitivity*, by Professor G. A. Hedlund. (Abstract No. 42-9-333-*t*.)

82. *A simplified solution of equation* $\Delta y = F(x)$, by Professor I. M. Sheffer. (Abstract No. 42-9-365-*t*.)

83. *A test-ratio test for continued fractions*, by Dr. Walter Leighton. (Abstract No. 42-9-340-*t*.)

84. *Complementary manifolds in* L_p *and* l_p, $1 < p \neq 2$, by Dr. F. J. Murray. (Abstract No. 42-9-352-*t*.)

85. *The integral in abstract linear topological spaces* (preliminary report), by Mr. E. W. Paxson. (Abstract No. 42-9-355-*t*.)

86. *On the uniform boundedness of sets of functional transformations between topological spaces*, by Professor L. M. Graves. (Abstract No. 42-9-330-*t*.)

87. *Free continuous groups.* II, by Mr. Garrett Birkhoff. (Abstract No. 42-5-241-*t*.)

88. *Lie groups isomorphic in the large with no linear groups*, by Mr. Garrett Birkhoff. (Abstract No. 42-5-240-*t*.)

89. *The enumeration of distributive lattices*, by Mr. Garrett Birkhoff. (Abstract No. 42-7-273-*t*.)

90. *Linear representability of nilpotent Lie algebras*, by Mr. Garrett Birkhoff. (Abstract No. 42-9-323-*t*.)

91. *The normal subgroups of the groups of matrices* mod p^r, by Dr. Joel Brenner. (Abstract No. 42-7-275-*t*.)

92. *On the theorem of Jordan-Hölder*, by Professor Oystein Ore. (Abstract No. 42-7-280-*t*.)

93. *On generalized Gauss sums*, by Professor Leonard Carlitz. (Abstract No. 42-7-312-*t*.)

94. *Hypercomplex numbers and factor sets*, by Professor Richard Brauer. (Abstract No. 42-9-324-*t*.)

95. *On algebraic equations of the fifth and sixth degrees*, by Professor Richard Brauer. (Abstract No. 42-9-326-*t*.)

96. *On the nth derivative of* $f(x)$, by Professor H. S. Wall. (Abstract No. 42-9-374-*t*.)

97. *On hypergroups*, by Professor H. S. Wall. (Abstract No. 42-9-373-*t*.)

98. *Concerning uniqueness-bases of finite groups with applications to regular p-groups of class* 2, by Dr. Charles Hopkins. (Abstract No. 42-9-334-*t*.)

99. *A normal form for the extension from a distributive lattice to a Boolean ring*, by Dr. H. M. MacNeille. (Abstract No. 42-9-342-*t*.)

100. *Dedekind's* ζ*-function and Hecke's* $\zeta(s, \lambda)$*-functions in totally real algebraic fields*, by Professor Hans Rademacher. (Abstract No. 42-9-359-*t*.)

101. *Ideal Waring theorem for the polynomial* $m(x^3-x)/6 -m(x^2-x)/2+x$, by Mr. Alvin Sugar. (Abstract No. 42-9-368-*t*.)

102. *Rational proofs of the existence of primes in arithmetic sequences*, by Dr. J. B. Rosser (National Research Fellow). (Abstract No. 42-5-238-*t*.)

103. *On regular closed curves in the plane*, by Professor Hassler Whitney. (Abstract No. 42-9-378-*t*.)

104. *Theory and applications of abstract normal coordinates in*

a general differential geometry, by Professor A. D. Michal and Mr. D. H. Hyers. (Abstract No. 42-9-348-*t*.)

105. *Abstract euclidean spaces. V: Rotations in spaces with indefinite geometrical metric*, by Professor A. D. Michal, Dr. I. E. Highberg, and Dr. A. E. Taylor. (Abstract No. 42-9-347-*t*.)

106. *A problem of replacement*, by Dr. Hillel Poritsky. (Abstract No. 42-9-356-*t*.)

107. *Thermal stresses in cylindrical tubes*, by Dr. Hillel Poritsky. (Abstract No. 42-9-358-*t*.)

108. *A symmetric reduction of the planar three body problem*, Professor F. D. Murnaghan. (Abstract No. 42-9-350-*t*.)

109. *Applications of Boolean algebras to general topology*, by Professor M. H. Stone. (Abstract No. 42-7-283-*t*.)

110. *Geodesic continua in abstract metric space*, by Professor Orrin Frink. (Abstract No. 42-7-292-*t*.)

111. *On boundaries in the plane*, by Professor G. T. Whyburn. (Abstract No. 42-7-306-*t*.)

112. *On continua of condensation*, by Professor G. T. Whyburn. (Abstract No. 42-7-305-*t*.)

113. *A property of hereditarily continuous curves*, by Mr. O. H. Hamilton. (Abstract No. 42-9-331-*t*.)

114. *Mappings and products of cell spaces*, by Professor A. W. Tucker. (Abstract No. 42-9-372-*t*.)

115. *Invariant theory of homogeneous contact transformations*, by Dean L. P. Eisenhart and Professor M. S. Knebelman. (Abstract No. 42-9-328-*t*.)

116. *Semi-continuity of integrals in the calculus of variations*, by Professor E. J. McShane. (Abstract No. 42-9-345-*t*.)

117. *Solvable groups*, by Mr. Philip Hall. (Abstract No. 42-9-389-*t*.)

118. *The web of quadrics*, by Professor T. R. Hollcroft. (Abstract No. 42-11-394-*t*.)

R. G. D. RICHARDSON,
Secretary

THE INTERNATIONAL CONGRESS IN OSLO

The International Congress of Mathematicians of 1936 was held in Oslo, Norway, July 13–18. The Congress was officially opened Monday evening, July 13, with a reception to the members of the Congress by the Rector of the University, in the festival hall of the University. The Rector was represented by Poul Heegaard, Dean of the Faculty of Science.

The inaugural meeting was held Tuesday morning, July 14, in the presence of H. M. the King in the festival hall of the University. The inaugural address was delivered by Professor C. Störmer, president of the organizing committee, and the official address of welcome was given by a representative of the government. The American delegation was particularly honored at this inaugural meeting in that two young men on the staffs of American universities were the recipients of medals offered by the late Professor Fields and which, according to the report of the Zurich Congress, are to be awarded to two mathematicians at intervals of four years by the International Congresses. The committee chosen at the Zurich Congress to name the recipients of the medals to be awarded in 1936 was composed of Professors Severi (chairman), Birkhoff, Carathéodory, Cartan, and Takagi. In the absence of Professor Severi, Professor Cartan took the chair. Professor Carathéodory spoke concerning the work of Professor Lars V. Ahlfors of Harvard University and Professor Jesse Douglas of the Massachusetts Institute of Technology, who were awarded the two Fields medals for outstanding achievement in mathematics.

Approximately five hundred mathematicians, representing thirty-two countries, were registered at the Congress. The total registration including families numbered over seven hundred. Of this number one hundred twenty-eight were from this country, and thirteen from Canada. The official delegates were Professors Eisenhart (chairman), Birkhoff, Blichfeldt, Lefschetz, Morse, Snyder, Veblen, and Wiener. The attendance included the following eighty-seven members of the American Mathematical Society:

L. V. Ahlfors, R. C. Archibald, H. T. R. Aude, I. A. Barnett, Harry Bate-

man, E. R. Beckwith, Garrett Birkhoff, G. D. Birkhoff, Wilhelm Blaschke, H. F. Blichfeldt, R. W. Brink, Thomas Buck, J. W. Campbell, Nancy Cole, J. T. Colpitts, A. H. Copeland, L. P. Copeland, Richard Courant, L. L. Dines, L. P. Eisenhart, H. T. Engstrom, Maurice Fréchet, Harald Geppert, M. C. Graustein, W. C. Graustein, M. R. Gut, Börge Jessen, S. A. Joffe, M. M. Johnson, B. de Kerékjártó, C. C. Krieger, M. N. Kryloff, Solomon Lefschetz, V. F. Lenzen, M. I. Logsdon, E. R. Lorch, A. J. McConnell, E. J. McShane, W. T. Martin, R. G. Mason, E. C. Molina, T. W. Moore, L. J. Mordell, E. M. Morenus, R. E. Moritz, Frank Morley, Marston Morse, Hermance Mullemeister, J. R. Musselman, E. P. Northrop, Rufus Oldenburger, Øystein Ore, C. W. Oseen, F. W. Perkins, Z. M. Pirenian, M. S. Rees, P. R. Rider, Grace Shover, L. G. Simons, C. E. Smith, D. E. Smith, P. F. Smith, T. L. Smith, Virgil Snyder, C. M. Sparrow, A. A. Stafford, M. H. Stone, J. L. Synge, Otto Szász, Olga Taussky, C. C. Torrance, H. W. Tyler, M. S. Vallarta, Oswald Veblen, J. H. Van Vleck, M. E. G. Waddell, G. N. Watson, Hermann Weyl, A. P. Wheeler, J. H. C. Whitehead, E. T. Whittaker, D. V. Widder, Norbert Wiener, W. L. G. Williams, John Williamson, A. H. Wilson, M. M. Young.

The following is a list of the invited addresses scheduled for the morning sessions of the Congress:

I. C. Störmer, *Programme for the quantitative discussion of electron orbits in the field of a magnetic dipole, with application to cosmic rays and kindred phenomena.*

II. R. Fueter, *Die Theorie der regulären Funktionen einer Quaternionenvariablen.*

III. E. Cartan, *Quelques aperçus sur le rôle de la théorie des groupes de Sophus Lie dans le développement de la géométrie moderne.*

IV. C. L. Siegel, *Analytische Theorie der quadratischen Formen.*

V. O. Veblen, *Spinors and projective geometry.*

VI. A. Gelfond, *Théories des nombres transcendants.*

VII. J. Nielsen, *Topologie der Flächenabbildungen.*

VIII. E. Hecke, *Neuere Fortschritte in der Theorie der elliptischen Modulfunktionen.*

IX. O. Neugebauer, *Über vorgriechische Mathematik und ihre Stellung zur griechischen.*

X. C. W. Oseen, *Probleme der geometrischen Optik.*

XI. V. Bjerknes, *New lines in hydrodynamics.*

XII. H. Hasse, *Über die Riemannsche Vermutung in Funktionenkörpern.*

XIII. G. D. Birkhoff, *On the foundations of quantum mechanics.*

XIV. S. Banach, *Le rôle de la théorie des opérations dans l'analyse.*

XV. L. J. Mordell, *Minkowski's theorems and hypotheses on linear forms.*

XVI. L. V. Ahlfors, *Geometrie der Riemannschen Flächen.*

XVII. J. G. van der Corput, *Diophantische Approximationen.*

XVIII. A. Khintchine, *Hauptzüge der modernen Wahrscheinlichkeitstheorie.*

XIX. M. Fréchet, *Mélanges mathématiques.*

XX. N. Wiener, *Tauberian gap theorems.*

XXI. Ø. Ore, *The decomposition theorems in algebra.*

Eight sections were organized for the afternoon meetings: (1) Algebra and Theory of Numbers. (2) Analysis. (3) Geometry and Topology. (4) Probability, Mathematical Statistics, Actuarial Theory, and Econometrics. (5) Mathematical Physics, Astronomy, and Geophysics. (6) Mechanics and Engineering. (7) Logic, Philosophy, and History. (8) Pedagogy. In section 2 there were three subdivisions and in section 3, two, making in all eleven groups meeting at the same time. The list of 15-minute papers presented at these afternoon section meetings by members of the American Mathematical Society either in person or by title is as follows:

R. C. Archibald, *New information concerning James Joseph Sylvester.*

I. A. Barnett and C. W. Mendel, *On an integral equation quadratic in the unknown function.*

H. Bateman, *Associated Airy functions in elasticity and hydrodynamics.*

Garrett Birkhoff, (1) *Generalized convergence*; (2) *Product integration of non-linear differential equations*; (3) *Order and the inclusion relation.*

A. H. Copeland, *Sequences with after-effect.*

R. Courant, *Über das Problem von Plateau.*

E. B. Cowley, *Function concepts in high school curriculums.*

H. Geppert, *Über den gemischten Inhalt zweier Bereiche.*

R. M. Gut, *Über Erweiterungen von unendlichen algebraischen Zahlkörpern.*

E. Kasner, (1) *Conformal and equilong symmetry*; (2) *Invariants conformes supérieures.*

B. de Kerékjártó, (1) *Topologie des transformations*; (2) *Sur la géométrie hyperbolique.*

M. N. Kryloff, *Les théories statistiques dans la mécanique non linéaire.*

E. J. McShane, *A non-absolutely convergent integration process.*

E. C. Molina, *Laplacian expansion for Hermitian-Laplace functions of high order.*

L. J. Mordell, *Note on the four integer cubes problem.*

R. E. Moritz, *A Napier theorem for quadrantal triangles.*

F. Morley, *Planar positions.*

M. Morse, *Functional topology and abstract variational theory.*

J. R. Musselman, *Circles connected with three or more lines.*

R. Oldenburger, *Non-singular multilinear forms and non-singular p-ic forms.*

F. W. Perkins, *Mean value theorems, with applications in the theory of harmonic, subharmonic, and superharmonic functions.*

M. Potron, (1) *Conditions des équilibres production-consommation et prix-salaires*; (2) *Irréductibilité de certaines intégrales abéliennes aux transcendants élémentaires.*

P. R. Rider, *Certain moment functions for Fisher's k-statistics in samples from a finite population.*

V. Snyder, *Certain Cremona transformations in S_n belonging multiply to a non-linear line complex.*

M. H. Stone, *Some remarks on linear functionals.*

J. L. Synge, (1) *On the connectivity of spaces of positive curvature*; (2) *Limitations on the behaviour of an expanding universe.*

O. Taussky, *Some problems of topological algebra.*

L. Tonelli, *Nouveaux problèmes dans le calcul des variations.*

C. C. Torrance, *Tangent lines and planes in topological spaces.*

M. S. Vallarta, *Results of calculations of asymptotic trajectories in the field of a magnetic dipole with applications to cosmic radiation.*

H. Weyl, *Faktorensysteme und Riemannsche Matrizen.*

J. H. C. Whitehead, *Equivalent sets of elements in a free group.*

D. V. Widder, *An integral equation of Stieltjes.*

The Proceedings will be published and sent to those participating in the Congress; these will contain the invited addresses in full and abstracts of the short papers.

On Tuesday afternoon, July 14, H. M. the King received the members of the Congress at the Royal Palace. Tea was served.

A bust of Sophus Lie was unveiled on Wednesday morning, July 15, by J. Sejersted Bödtker, who represented the donors of this gift to the University.

On Wednesday evening the City of Oslo gave a dinner for the ordinary members of the Congress at the Hotel Bristol. Direktor Gann was master of ceremonies, and Disponent Stranger delivered the address of welcome. Professors Eisenhart, Schmidt, and Julia spoke representing the English, German, and French speaking members respectively, and Professor Störmer gave the formal "takke tale."

One of the most enjoyable occasions of the Congress was the excursion on the Oslofjord Thursday afternoon, July 16, aboard the transatlantic steamer "Stavangerfjord." H. R. H. the Crown Prince, Honorary President of the Congress, and H. R. H. the Crown Princess graciously consented to take part. Dinner was served on board. Professor Schieldrop, Secretary of the Congress, was master of ceremonies. H. R. H. the Crown Prince spoke. The other speakers were Professor Störmer and M. Emile Borel. Dancing followed. The rolling hills and mountains of the Oslofjord were particularly lovely in the twilight which lasted until midnight, the hour of return.

On Friday, July 17, the American Embassy at Oslo gave a luncheon in honor of the American delegates and the members of the directing committee of the Congress.

An extremely interesting program was planned especially for the ladies. This included a visit to the Norwegian Folk Museum and the Viking ships at Bygdóy, a visit to the National Gallery, a luncheon, a tea, a dinner, and several motor trips in the vicinity of Oslo.

At the closing meeting of the Congress held on Saturday, July 18, the invitation to hold the next International Congress in America in 1940 was accepted with thanks.

The entire program of the Congress showed careful planning on the part of the Norwegian committees, and it was executed with smoothness and precision. The American members of the Congress were unanimous in their appreciation of the warm hospitality of the Norwegian people and mathematicians.

MARSTON MORSE

ALEXANDROFF AND HOPF ON TOPOLOGY

Topologie I. By Paul Alexandroff and Heinz Hopf. Berlin, Springer, 1935. xiii+636 pp.

Topology is not a young subject; this year may be described as the two hundredth anniversary of its birth if we agree that it had its beginning in the problem of the seven bridges of Königsberg settled by Euler in 1736. But the systematic development of topology is new; it has only come since the work of Poincaré at the turn of the century. The International Topological Conference held at Moscow last September showed that the subject has attained a definite measure of maturity and a wide range of influence on other branches of mathematics, but that it is still undergoing rapid growth and flux. Just when topological activity seems to be slackening some new point of view sets it seething again; in the past year we have had an example of this in the "dual cycles" of Alexander and Kolmogoroff.

Books on topology are so few that the appearance of a new one is an important event. The volume now before us is especially impressive, not only for its own size and thoroughness, but because it is the first of three volumes which together are intended to give a detailed survey of topology as a whole. The authors are distinguished geometers who have been in close contact with the various topological centers in Europe and America. They began their difficult task several years ago at the suggestion of Courant, under whose editorship the book appears as the forty-fifth of the familiar Springer series. Many of us have known about the project and have expectantly awaited the finished work.

In their attempt to treat topology as a whole the authors have steered a middle course between point-set theory on the one hand and algebraic topology on the other. They do this quite easily since in their own investigations they both have used combinatorial methods to solve set-theoretic problems; no better example of such blending could be cited than Alexandroff's theory of dimension. As the central concept for the first volume they have chosen the (euclidean) *polyhedron*, a subset of multi-dimensional euclidean space which can be partitioned into flat convex cells in an orthodox manner. The polyhedron holds in their program an intermediate position between the generality of an *abstract* space and the special character of a *manifold;* by appropriate approximation, for example, one passes from polyhedra to compact metric spaces, and by suitable specialization one passes from a polyhedron to a manifold. In accordance with this plan the second volume is to deal with set-theoretic questions, such as dimensionality, and the third with manifolds.

The book (that is, the first volume) falls into four parts. Part I is a hundred-page introduction to abstract-space topology; much of this has no bearing on the subsequent study of polyhedra but it does have an essential place in the general program of the three volumes. The first chapter begins with various methods of topologization, closure, limits, distance, neighborhoods, and so on, then passes to separation and countability axioms, and finally to the Urysohn theorem that a normal space with countable basis can be immersed in Hilbert

space. The second chapter treats the fundamental properties of compact and bicompact spaces. The authors exploit to interesting advantage the notion of a decomposition of a space into mutually exclusive closed sets.

In Part II we encounter the fundamental machinery of combinatorial topology set up on abstract simplicial complexes, finite or countably infinite. Chapter 3 introduces polyhedra and their subdivisions into convex cells as a concrete basis from which to pass to abstract complexes. Chapters 4 and 5 develop the homology theory of absolute (abstract) complexes in strict group terms, the theory of Abelian groups prerequisite for this being given in an appendix. For chains and cycles the authors use coefficients drawn from a general Abelian (additive) group, assuming only when necessary that the coefficient group is a ring. Moreover they introduce two coefficient groups at once, one group for cycles and a second one, which contains the first, for homologies; this is a generalization of cycles with integer coefficients which do not bound even with division allowed. Detailed consideration is given to the special properties of homology over the rings of integers, of rationals, and of residues mod m, and over the Pontrjagin group of rationals mod 1, which is proving such a useful idea nowadays. The invariance of homology under subdivision is established. Chapter 6 discusses the consolidation of simplexes into more general combinatorial units and relates it to the subdivision of polyhedral complexes. A final chapter (7) deals with closed complexes (circuits) and with the homology theory of sums and (cartesian) products of complexes.

In Part III we return to (euclidean) polyhedra to begin a comprehensive investigation of the topological invariants which attach to them out of the homology theory now established. Chapter 8 uses simplicial approximation of continuous mapping to collect homotopic mapping-classes into homology classes and so to establish in large measure the topological invariance of homology; the same chapter also introduces continuous (singular) chains and cycles, and the notion of retract. The main proofs of the invariance of homology groups and of dimension are formulated in Chapter 9 in terms of coverings and nerves; much of this applies to compact metric spaces as well as to polyhedra (including infinite polyhedra). Chapter 10 has to do with the decomposition of a euclidean space by a compact metric subspace, the Jordan-Brouwer theorem, and the invariance of open regions.

Part IV continues the application of homology theory to the study of polyhedra—first to the relative position a polyhedron may have in euclidean space, and then to properties of mappings of polyhedra into themselves or into euclidean spaces or spheres. The question of relative position is handled in Chapter 13 by the theory of linking and the Alexander duality theorem. Chapter 14 deals with the Brouwer concept of the degree of a mapping, and with outgrowths thereof. In Chapter 15 we have a discussion of the extension of mappings in euclidean space and of the types of maps of an n-dimensional polyhedron on the n-sphere or on the circle. A very full account of the Lefschetz fixed-point formula as set up for polyhedra by Hopf occupies the last chapter (16). There are two appendices, one on Abelian groups and the other on convex cells in euclidean space.

The authors have carried through the above program with great completeness of detail; they have explored many bypaths, and in some instances they

have given alternative demonstrations for basic propositions and alternative approaches to important concepts. Unfortunately this extreme comprehensiveness coupled with a tendency to be somewhat discursive renders the book formidable for beginners and not exactly suited to straightforward reading. But a reader with some topological experience should be able to make his way about in the book and benefit from its encyclopaedic nature without being overwhelmed. He will find his task facilitated by a detailed table of contents at the beginning of each chapter, a good general index, and abundant cross-references (but not to page numbers, unfortunately). There are also many helpful figures and examples.

The book contains no real bibliography; it has merely a list of topological texts and one of works which bear directly on individual sections of the book and influenced their composition. References to the literature through footnotes have been reduced to a minimum. Certain concepts and proofs have been designated by the names of their originators, but otherwise the authors have resolutely refrained from attempting to trace notions back to their sources. The introduction to the book embraces a short history of the development of topology and a survey of the relations of topology to neighboring branches of mathematics, as well as a discussion of the authors' program. Very appropriately the book is dedicated to L. E. J. Brouwer.

A. W. TUCKER

MILLER'S COLLECTED WORKS

The Collected Works of G. A. Miller. By H. R. Brahana, R. D. Carmichael, and A. B. Coble (the committee on publication), and G. A. Miller. University of Illinois Press. Volume 1, 1935. xi+475 pp.

In the preface written by the committee on publication, consisting of H. R. Brahana, R. D. Carmichael, and A. B. Coble, of the University of Illinois, it is stated that "the preparation of this volume is almost wholly the work of Professor Miller." This volume contains Miller's contributions up to the year 1900 together with three papers (numbered 1, 15, 62) written for the purpose of showing the place of this work in the theory and to give a brief survey of the historical development of the subject up to 1900.

The papers in this volume with exception of 1, 15, and 62, are arranged substantially in the order in which they were published, but for the purpose of exhibiting the nature of the work contained in them it will be convenient to group them under the following headings: (1) Papers which relate to the determination of groups of finite order. (2) Expository papers on the construction of permutation groups; applications. (3) Papers which deal with the theory of groups of finite order. (4) Paper on recent progress; miscellaneous papers. These will be considered in the order given.

(1) *Papers which relate to the determination of groups of finite order.*

These are as follows: (a) Permutation groups of given degree; (b) Abstract groups of given order; and (c) Non-regular permutation groups of given order.

(a) *Permutation groups of given degree.*

To give a proper introduction and setting to his early papers on the listing of permutation groups of given degrees, Miller has written a special paper (numbered 1) entitled *Historical note on the determination of all permutation groups of low degrees.* In this paper there is given a clear account of enumerations made by earlier writers along with a brief statement of the methods used.

Including the historical note just mentioned there are fifteen papers devoted to the listing of permutation groups of given degrees. The most important of these papers are devoted to the determination of all permutation groups through degree ten, to the determination of all transitive groups of degrees twelve, thirteen, and fourteen, and to the finding of all primitive groups of degrees fifteen, sixteen, and seventeen. Lists of the intransitive groups of degree ten and of the imprimitive groups of degrees thirteen and fourteen were first published by Miller. Other papers are concerned with the correction of errors in lists published by earlier writers. Errors were found in all degrees from eight to seventeen, inclusive, with the exception of degrees eleven, thirteen, and fourteen. To find these errors it was necessary to make complete lists for all degrees examined by earlier writers, and in the early stages when the theory upon which this work is based was not far developed, this involved a considerable amount of tentative work. Later determinations, which were based wholly or in large part on theory, show that his earlier com-

putations were entirely accurate. His important memoir on the substitution groups whose degrees do not exceed eight, and his determination of the primitive permutation groups of degree ten are based entirely on theoretical considerations. For his work on the permutation groups of degree ten he was awarded in 1900 a prize by the Cracow Academy of Sciences.

(b) *Abstract groups of given order.*

To provide an introduction to his early papers on abstract groups, Miller has written a special paper (numbered 15) entitled *Historical note on the determination of abstract groups of given order*. This note is an amplification of a similar paper published in the Journal of the Indian Mathematical Society in 1932. In this paper there is given some account of postulational definitions of finite abstract groups, and a sufficient account of the work that has been done on the determination of abstract groups of given order to afford a proper setting for the author's work in this field. It also gives a brief statement of the work by the author on a problem initiated by him, namely, to determine all orders for which a given number of abstract groups exist.

In addition to the historical note just referred to, there are five papers in this volume devoted to the determination of all abstract groups of given order. One of these gives a complete determination of all abstract groups whose orders are less than forty-eight. In this paper the groups of orders sixteen, twenty-four, thirty-two, and forty are treated in some detail. The groups of other orders less than forty-eight follow readily from earlier considerations. The paper includes also some remarks on regular groups and the earliest published development of the properties of commutator sub-groups, the usefulness of which was widely recognized later.

The abstract groups of order $8p$ where p is any prime number are determined in a second paper. In this paper consideration is given first to those groups which contain an invariant sub-group of order p, and second to those groups which do not contain such a group. In this and the preceding paper the method used is that of constructing imprimitive permutation groups of given degree. A third paper is devoted to the determination of all abstract groups whose orders are greater than forty-seven and less than sixty-four, and of those whose orders are $2p^3$ where p is any prime number. In this paper the groups of order forty-eight are given particular attention and those of order fifty-four are obtained as a special case of the groups of order $2p^3$. The other groups whose orders are less than sixty-four and greater than forty-seven follow readily from determinations made by earlier writers. Other papers which relate to abstract groups of given orders consist largely of brief reports published in the Comptes Rendus on lists of these groups. One of these gives also some information on permutation groups whose degrees are less than thirteen.

(c) *Non-regular permutation groups of given order.*

In addition to the papers which relate to the determination of all permutation groups of given degrees and of all abstract groups of given orders, Miller has written several papers which relate to the determination of permutation groups of given orders and to the number of transitive permutation groups which are isomorphic to a given group. In this volume there is one paper on the permutation groups of order four and another on the permutation groups

whose orders are the product of two distinct prime numbers. In other papers determinations of the non-regular transitive permutation groups of orders p^3, pqr, and $8p$ are made, where p, q, r are prime numbers. Two papers treat the number of transitive permutation groups isomorphic to a given group. One of these relates to the transitive permutation groups simply isomorphic to the symmetric or to the alternating group of degree six; the other to the number of transitive permutation groups that have a 1, ∞ isomorphism to a given group.

(2) *Expository papers on the construction of permutation groups; applications.*

There is included in this volume a series of three expository papers (numbered 9, 10, 11) which were published in volumes 2 and 3 of the American Mathematical Monthly under the titles: *Remarks on substitution groups*, *Introduction to substitution groups*, and *Applications of substitution groups*. The first of these presents a few basic concepts in the theory of substitution groups and lists the substitution groups of degrees two and three. The second paper develops in a simple manner the elementary methods of constructing permutation groups and applies these methods to the construction of all permutation groups whose degrees do not exceed six. This paper is quite valuable since it gives in simple form what is perhaps the best discussion to be found in the literature of methods for finding permutation groups of given small degrees. The third paper is devoted to the study of some of the relations which exist between substitution groups and functions that involve a given number of variables. Applications are then made to determine the number of different formal values of the cross-ratios of four points under the symmetric group of degree four, to find the other formulas in trigonometry which correspond to that for the sine of a given half-angle of a triangle in terms of the sides, and to a discussion of a simple problem in factoring. Another paper published in the Monthly is devoted to a practical application of a substitution group in spherical trigonometry. Three additional papers also published in the Monthly, which are largely expository in character, treat the groups determined by a given group, a method for constructing intransitive permutation groups, and the presentation in a very simple manner of some elements of substitution groups.

(3) *Papers which deal with the theory of groups of finite order.*

These papers may be classified into those which relate to the properties of abstract groups and those which relate to the properties of permutation groups. Those on abstract groups deal, for the most part, with properties of commutators and commutator groups, Hamilton groups, perfect groups, and the quaternion group. Important properties of each of these special classes of groups are developed by Miller. His work on commutator sub-groups merits special attention since the earliest published properties of these groups were given in his paper on the regular substitution groups whose orders are less than forty-eight which was published in the Quarterly Journal of Mathematics, volume 28 (1896). In this paper application of these properties was made to test the solvability of the groups found. Two later papers are devoted to further study of commutators and commutator sub-groups. Other papers devoted to abstract groups deal with an extension of Sylow's Theorem and with properties of groups of order p^∞ where p is a prime number.

Several general theorems on permutation groups are proved by Miller in

his papers which relate to lists of permutation groups of given degrees and of abstract groups of given orders. For example, his paper in volume 28 (1897) of Proceedings of the London Mathematical Society on primitive substitution groups of degree fifteen contains the proofs of several theorems which relate to the maximal sub-group of degree $n-1$ in a simply transitive primitive group of degree n. Additional properties of these sub-groups are developed in a brief paper published later in this Bulletin. Two papers on permutation groups determine conditions under which permutation groups of given special degrees contain simple groups. Two additional papers are devoted to multiply transitive groups. One of these includes the proof of a theorem on an upper limit of transitivity and the other gives a necessary and sufficient condition that a group may be represented as a multiply transitive group. Other papers which relate to permutation groups are somewhat more elementary in character. One of these gives a new proof of the well known theorem on the average number of letters in a permutation group of degree n; a second relates to the matter of definition of distinct permutation groups; and a third deals with the simple isomorphisms of a group to itself.

(4) *Paper on recent progress; miscellaneous papers.*

A paper entitled *Report on recent progress in the theory of groups of a finite order* was published in Volume 5 (1899) of this Bulletin. It presents developments of importance in this field during the five preceding years which were not fully treated in existing treatises. The developments are presented under the headings solvable groups, simple groups, substitution groups, and abstract groups. Attention is called also to unsolved problems of interest.

Miscellaneous papers not included in what precedes present Sophus Lie's views on several important points in modern mathematics, some reminiscences in regard to Sophus Lie, several points in the theory of groups of a finite order (this being a paper read before the Oliver Mathematics Club, Cornell University), and a note on Burnside's *Theory of Groups*. In the last note attention was called to several errors in the first edition of this treatise.

The last paper in the volume (numbered 62) is entitled *History of the theory of groups up to* 1900. Along with the other two papers (numbered 1 and 15) written for this volume by Miller, it shows the connection between his work before 1900 and the general developments in the subject, and it gives a suitable basis for seeing the connection between these early papers and his later work.

In this paper the history of the theory of groups up to 1900 is divided into five periods, as follows: (1) Developments antedating the beginning of the nineteenth century. (2) The first forty years of the nineteenth century. (3) The period from 1840 to 1860. (4) The period from 1860 to 1880. (5) The years 1880 to 1900. Excellent brief accounts are given of all important developments in group theory during each of these five periods with the exception of the important work of G. Frobenius toward the end of the last period on group characters. Reference to this work of Frobenius is given, however, in the paper (numbered 48) on recent progress. Much valuable information is also given on the influences in the various countries which led to these developments. This important paper will be of value to many students of group theory.

H. W. KUHN

SHORTER NOTICES

Les Applications des Probabilités. By P. Van Deuren, Paris, Gauthier-Villars, 1935. xvi+556 pp.

This interesting volume on applications of probability is a continuation of Van Deuren's *Théorie des Probabilités*, which was reviewed earlier in this Bulletin.* Retaining the formal or "didactic" mode of presentation, the book deals with statistics, errors of measurements, play or speculation, life insurance, artillery fire, the "normal numbers" of Borel and continued fractions, statistical mechanics, astronomy, telephone problems—with appendices on the Tchebychef theorem and correlation. The references to other writers are few, especially in the earlier chapters where the material is made up largely of the standard or classical applications of probability from the standpoint of moments. The treatment of these problems, however, is individualistic, the author using the conceptions of *concentration* of variables and *practical certainty* as developed in his volume on the theory of probability.

In the theory of speculation there is discussed not only the plays that are equitable, but those with a slight advantage to the banker; and a parabola (p. 274) exhibits graphically the banker's risk. This is a natural prelude to the discussion of the loading for expenses in life insurance policies. The author refers to the Institute of Actuaries Text-Book, by G. King, and to some extent shows the connection between his own notation and that of the Text-Book. It might have been well to call attention to the fact that the author's fundamental *moment statistique*, r, is the complete expectation of life at age zero (p. 331). The constants for the Makeham graduation in the Text-Book are given (p. 341), and two methods are suggested for graduation—neither of which is that used in the Text-Book. In the chapters on life insurance, the reader should notice that the author is dealing primarily with payments made continuously, and that in the case of annual payments (p. 368) the first payment is due six months from date. Commutation symbols here, therefore, do not have their usual interpretation. To find the adequacy of the loading, the author suggests tests not usually made by insurance companies (pp. 386, 394).

Of special interest are the chapters on artillery fire (pp. 411–480) which the author as Professeur à l'École Militaire de Belgique is so well prepared to write, giving in detail material not usually found in books on probability.

In connection with the treatment of the *normal numbers* of Borel, the author exhibits the first five hundred decimals of π, and comments on the somewhat spectacular shortage in the sevens. The chapters on statistical mechanics and telephone problems are attributed to Lieutenant du Génie R. Marchant. In the former are discussed the Maxwell laws, Heisenberg's principle of indeterminacy, the theory of Bose-Einstein and that of Fermi-Dirac. The chapter on astronomy is attributed to R. Hennet.

In many ways, Van Deuren's book is unique; and it is well worth reading.

E. L. DODD

* Dodd, vol. 42 (1936), pp. 16, 17.

The Vectors of Mind. By L. L. Thurstone. (The University of Chicago Science Series.) University of Chicago Press, 1935. xv+266 pp.

It has been found by psychologists attempting to "explain" the correlations resulting from a set of tests, that the original Spearman hypothesis of a single common factor is generally untenable. The natural extension of the hypothesis is that the correlations can be explained by several common factors operating linearly. During the last few years this general hypothesis has attracted the attention of several investigators, particularly Hotelling, Kelley, and Thurstone. In this book, Professor Thurstone has given a detailed exposition of methods which he has developed for handling the multiple factor problem. He has found the theory of matrices to be an effective tool for treating the problem and has included a lengthy introduction on elementary matrix theory and linear transformations required for the subsequent pages.

Expressed in its simplest form the fundamental problem is essentially that of finding a matrix F containing as many rows as traits measured, and having the least number of columns and greatest number of zero elements (subject to various restrictions) such that, except for sampling variations, $FF'=R$, where F' is the transpose of F, and R is the matrix of correlation coefficients with variances due to common factors, as diagonal elements. The first two chapters are devoted to an elaboration of the nature of this problem and its mathematical formulation. By considering each trait as a point in the common factor space, a geometrical interpretation of the factor problem is developed along with its analytical treatment. In fact, most of the definitions are written in geometrical language.

A procedure for factoring R, called the centroid method, is developed in Chapter 3. This method consists of finding the elements of F from the matrices of several sets of transformations of the common factor space, each set being a cycle of rotations, translations, and reflections, designed to locate the centroid of a corresponding "residual configuration" of trait points. A method for finding an F from the orthogonal transformation associated with the principal axes is given in Chapter 4. This method is similar to Hotelling's method of principal components. In Chapters 3 and 4 the author deals mainly with the problem of finding an F with a minimum number of columns, so that, except for sampling variations, $FF'=R$. In Chapters 6–10, he is concerned with isolating primary traits, that is, a further trimming of F by transforming to new coordinates in the common factor space in such a way as to get (ideally) a maximum number of zero elements in F. Chapter 11 is a discussion of the regression method of appraising the primary traits in each individual. The book is well equipped with applications of the method to real as well as fictitious data. Several appendices of laboratory directions are given which are extremely valuable to those interested in applying the centroid method or the method of principal axes to actual data.

The author has presented an enlightening discussion of the multiple factor problem and methods for handling it. However, as many, if not more, problems are raised than solved, which is often inevitably true of works in a new field. The author scarcely mentions the sampling aspect of the multiple factor problem, which is a matter of considerable importance in an application of the

method to as few as two hundred and forty individuals, (the number in a study reported by the author). For this reason the generality which he claims by not assuming normality is, for the most part, illusory. Of prime importance among the mathematical problems are those which arise in dealing with sampling variations. These would include the problem of determining optimum estimates of the elements in F, and in actually developing an objective criterion and its probability theory for testing the hypothesis that a set of correlation coefficients can be explained by an F which has been found by any method whatever. Further development of the multiple factor theory will depend largely upon solutions of these problems.

S. S. WILKS

Leçons d'Analyse Vectorielle. By Gustave Juvet. Part II. Paris, Gauthier-Villars, 1935. 306 pp.

This second part of Juvet's *Vector Analysis* contains the applications to mathematical physics. These relate mainly to potential theory, fluid dynamics, and electromagnetic theory. The book contains a good collection of exercises. To provide a basis for the fundamental existence theorems, a brief treatment of Fredholm's theory of linear integral equations is given and an appendix is included which contains the elements of the theory of functions of a complex variable. The mathematical discussions are of intermediate character, making use of conditions of continuity and convergence, but not stressing these matters as much as is customary in works on pure mathematics.

H. B. PHILLIPS

Science and the Human Temperament. By Erwin Schrödinger. New York, W. W. Norton, 1935. 24+192 pp.

Schrödinger writes: "The old links between philosophy and physical science . . . are being more closely renewed. The farther physical science progresses the less can it dispense with philosophical criticism." This book may be regarded as a substantiation of this thesis by means of an illuminating analysis of certain fundamental ideas and issues in contemporary physics.

The most important chapter is entitled *The Fundamental Idea of Wave Mechanics.* It presents the author's theory as a natural development of, and the first theoretical justification for, the similarity between Fermat's principle of minimum time in optics and Hamilton's minimum principle in mechanics. This and the entire previous chapter discuss the role of models in physics.

Two other philosophical issues receive extensive consideration. They are scientific law and causality. The universal reduction of statistical to causal laws is regarded as unjustified. Even the rigorous application of causality in Newtonian mechanics is queried. It required that velocity be determined in defining a state. But velocity was identified with a differential quotient which was defined as $(x_2-x_1)/(t_2-t_1)$ as $t_2-t_1\to 0$. Hence, the velocity referred "to two units of time and not the state at one moment." Against the coincidence of the two units in the limit, he adds that "possibly this mathematical process of approach to the limit . . . is inadmissible" and "is inadequately adapted to nature" (pp. 61–62).

This suggestion from physics becomes more interesting when we note in pure mathematics that the rigorous formulation of the calculus necessitated assumptions by Dedekind and Cantor which lead to unresolved contradictions. Similar considerations have suggested to this reviewer that the theoretical difficulties at the basis of physics and mathematics may have much more in common than has been realized, and that a clue to their resolution may be found in an alteration in the more general philosophical assumptions common to the two sciences.

F. S. C. NORTHROP

L'Arithmétique dans les Algèbres de Matrices. By Claude Chevalley. (Actualités Scientifiques et Industrielles, No. 323.) Paris, Hermann, 1936. 33 pp.

This book is one of the collection called *Exposés Mathématiques*, published in memory of the late Jacques Herbrand.

This investigation represents another step in the simplification by abstraction of the number theory of linear algebras, a theory which seemed so impossibly complicated when it was first attacked but a few years ago. Ideal theory has grown in importance until, as in the present paper, it constitutes the whole of arithmetic.

Let $\mathfrak{S}$ be a ring with unit element in which every regular element (that is, not a divisor of zero) has an inverse. Then $\mathfrak{S}$ has a regular arithmetic when there is defined a system of modules $\mathfrak{A}, \mathfrak{B}, \cdots, \mathfrak{O}, \cdots$, called ideals such that:

I. The ideals form a groupoid under modular multiplication. The left (right) order of an ideal $\mathfrak{A}$ is the totality $\mathfrak{O}(\mathfrak{O}')$ of elements $\lambda(\lambda')$ such that $\lambda\mathfrak{A} \subset \mathfrak{A}(\mathfrak{A}\lambda' \subset \mathfrak{A})$. The inverse $\mathfrak{A}^{-1}$ of $\mathfrak{A}$ is the set of elements μ such that $\mu\mathfrak{A} \subset \mathfrak{O}'$, $\mathfrak{A}\mu \subset \mathfrak{O}$.

II. If $\mathfrak{A}$ is an ideal, every element of $\mathfrak{S}$ is a product of an element of $\mathfrak{A}$ by the inverse of a regular element of $\mathfrak{A}$.

III. If $\mathfrak{O}$ is a unit of the groupoid, the ideals which have $\mathfrak{O}$ for their left order are the finite left $\mathfrak{O}$-modules which contain regular elements.

IV. In every class of left or right ideals there is an integral ideal prime to any given integral ideal.

The principal result obtained is that if $\mathfrak{S}$ is a total matric algebra over a (not necessarily commutative) field k in which a regular arithmetic is defined, one can define a regular arithmetic in $\mathfrak{S}$. It is known that such a regular arithmetic can be defined in every simple algebra whose centrum is an algebraic field. We see the importance of this result if we recall Wedderburn's theorem that every simple algebra is a total matric algebra over a division algebra.

The above theorem leads to further results in the theory of ideal classes in $\mathfrak{S}$.

C. C. MACDUFFEE

Variationsrechnung und partielle Differentialgleichungen erster Ordnung. By C. Carathéodory. Teubner, Leipzig and Berlin, 1935. 11+407 pp.

A century ago Jacobi, influenced by Hamilton's work on geometrical optics, discovered that there exists a direct connection between the theory of the calculus of variations and the theory of partial differential equations of the

first order. Since that time many brilliant mathematicians have contributed to the development of the calculus of variations and some have noticed the connection announced by Jacobi. But it has remained for Professor Carathéodory to give a systematic account of the subject in the book under review.

The book is divided into two parts. The first part (163 pp.) is devoted to partial differential equations of the first order and is an elegant presentation of the theory which is necessary to bring out the relationships with the calculus of variations, which forms the second part of the book. The author's treatment of the simplest problem of the calculus of variations leads directly to the Hamilton-Jacobi equations of mechanics and the foundations of the subject are fully discussed.

The author has not attempted to give a complete treatment of the later developments but has intended to carry them far enough to connect with existing literature. For this purpose the bibliography, containing 190 references to books and articles, is followed by a very helpful section giving advice on the use of the literature.

To quote from the preface: "The purpose which I have pursued in this book will have been accomplished if the student of this field of mathematics may become convinced that there are today three principal aspects of the calculus of variations: first, the calculus of variations of Lagrange, which today forms a part of the tensor calculus; second, the theory of Tonelli, in which the finer aspects of the minimum problem are based on point set theory; then the point of view presented in this book, which is oriented to the theory of differential equations, differential geometry, and physical applications, which was first made prominent by Euler. I hope also to have shown that the Weierstrass theory belongs to this last aspect."

Professor Carathéodory is not only a master of his subject but also a master of presentation. This clearly written fundamental work will prove indispensable for all students of the subject.

W. R. LONGLEY

Introduction to the Theory of Linear Differential Equations. By E. G. C. Poole. London, Oxford University Press, 1936. 200 pp.

This book deals with ordinary linear differential equations and is based on lectures delivered by the author to senior undergraduates at Oxford. Natural prerequisites are an elementary course in differential equations and considerable familiarity with the theory of matrices. The selection of material is such as to form an excellent introduction to this vast and important field of mathematics.

The first half of the book deals with properties common to wide classes of equations. The first chapter, on existence theorems, presents the fundamental theorems for given initial conditions. In the second chapter the solutions of equations with constant coefficients, subject to initial conditions of the Cauchy type, are obtained by the methods of the Heaviside operational calculus. The third chapter is concerned with some formal investigations involving linear operators, adjoint equations, and simultaneous equations with variable coefficients. The next two chapters deal with equations having uniform analytic coefficients, attention being focused on regular singularities. In the second half

of the book the author chooses for detailed study the hypergeometric equation, Laplace's linear equation, and the equations of Lamé and Mathieu.

The style of the book is somewhat terse and, while usually clear, is not always easy reading for one approaching the subject for the first time. This difficulty is largely overbalanced by abundant references to the literature so that an industrious reader will be able to learn very much with this volume as a basis for his study. The details given in the text are considerably extended by the collection of examples at the end of each chapter. A large number of these examples contain citations to their origin in the literature.

W. R. LONGLEY

La Théorie du Potentiel et ses Applications aux Problèmes Fondamentaux de la Physique Mathématique. By N. M. Gunther. Paris, Gauthier-Villars, 1934. 303 pp.

This volume is one of the collection of monographs on the theory of functions published under the direction of Émile Borel and contains a carefully written and rigorous treatment of the material usually covered in an introductory course on the theory of the newtonian potential function.

The first chapter contains general definitions and theorems concerning functions of the type to be encountered later. The characteristic properties of the newtonian potential of a three-dimensional distribution of attracting matter and of a simple and of a double surface distribution are treated in the second chapter. The remaining three chapters are concerned with the standard problems associated with the names of Neumann, Robin, Dirichlet, and Green.

The book is entirely self-contained in that it contains only one reference to the literature and this occurs in the last paragraph of an appendix.

W. R. LONGLEY

Le Problème de la Dérivée Oblique en Théorie du Potentiel. By G. Bouligand, G. Giraud, and P. Delens. Paris, Hermann, 1935. 78 pp.

This is No. 219 of the series *Actualités Scientifiques et Industrielles* and No. 6 of the subseries devoted to geometry and edited by E. Cartan. The problem treated is the extension of the Neumann problem in potential theory where the normal derivative is replaced by a directional derivative whose direction is prescribed over the bounding surface. When the direction is never tangent to the bounding surface and when, in addition, the direction cosines fixing the direction and the values assigned to the directional derivative satisfy certain regularity conditions (Hölder and continuity conditions) the problem is termed regular. The first part of the present work, written by Bouligand, is introductory and shows the essential character of the criterion of regularity (when the direction of differentiation can become tangent to the bounding surface the uniqueness theorem which holds in the regular problem breaks down). The second part, written by Giraud, gives the solution of the regular problem (under a stated condition of compatibility on the assigned values of the directional derivative). The third part, by Delens, discusses the connection of the theory of congruences of curves with the problem. This arises through a study of harmonic functions of the form $\phi(\alpha, \beta)$, where α, β are functions of position.

F. D. MURNAGHAN

Collected Papers of Charles Peirce. Vol. III, *Exact Logic*, xiv+433 pp., and Vol. IV, *The Simplest Mathematics*, x+601 pp. Edited by Charles Hartshorne and Paul Weiss. Harvard University Press. 1933.

Nearly all the papers in Volume III of this series have been previously published, and the most important of these have been summarized and evaluated by C. I. Lewis in his *Survey of Symbolic Logic*. The volume is devoted almost wholly to a development of the logic of relations and to a discussion of the bearing of this logic upon mathematics and upon the analysis of propositions of ordinary discourse. There are two papers in which Peirce discusses parts of Schröder's *Vorlesungen über die Algebra der Logik*.

Volume 4 comprises unpublished papers on logic and mathematics. Although of less intrinsic value than those of the preceding volume, they are perhaps of more interest now because of the wealth of brilliant suggestions they contain. The volume is divided into three parts. The first contains miscellaneous papers on logic, on the foundations of arithmetic, and on Cantor's transfinite numbers; the second is devoted to Peirce's theory of *graphs*; while the third contains an exposition of a number of mathematical curiosities.

The editors call special attention to an unpublished paper (vol. IV, pp. 13 ff.) written about 1880, which, as they point out, involves a clear anticipation of Sheffer's stroke-function. Peirce writes AB to mean "not-A and not-B." So AA means "not-A," and the other definitions follow as with Sheffer. Later on (p. 216) Peirce points out how the same definitions can be given if AB is read "not-A or not-B," which is the other interpretation Sheffer puts upon his stroke-function. Peirce says that the single function AB can replace the seven symbols $=$, $>$, $+$, $-$, $\times$, 0, 1, which are the constants of Boole's calculus of classes. It looks, however, as if this claim could not be allowed. If we give the symbols the propositional interpretation, then there will be nothing in the functions constructed on AB which represents a class; if we give the symbols the class interpretation, then, in order to carry through all the definitions as given by the editors in a note on page 18, we shall have to go over occasionally to the propositional interpretation. It is the propositional interpretation to which the scheme fully applies.

The second part of Volume IV contains, as we have said, an exposition of Peirce's system of graphs, which are to represent any propositions (after the fashion of the Euler diagrams) and to represent argument or proof by rules which govern transformations of these graphs. Peirce held that all formal proof (logical and mathematical) consists in reading off facts from a diagram of some form or other; such proof is pictorial or *Iconic* (see p. 430). He therefore places much emphasis upon formal algebras and graphical representation.

In the course of developing his graphs, Peirce tries in several places to supply graphical representations of modal propositions and analytical or necessary connections of propositions (see esp. pp. 398 ff.) such as those subsequently dealt with by C. I. Lewis in his *Strict Implication*. But Peirce's conception of modality differs in an important respect from that of Lewis: he makes possibility and necessity relative to states of information. For this reason the comparison which the editors draw in a footnote on page 403 between Peirce's diagrams and propositions of Lewis's system cannot be exact.

C. H. LANGFORD

L'Arithmétique de l'Infini. By Maurice Fréchet. Paris, Hermann et Cie, 1934. 38 pp.

Propriétés des Espaces Abstraits les plus Généraux. Ensembles Ouverts, Fermés, Denses en soi, Clairsemés. Connexion. By Antoine Appert. Paris, Hermann et Cie, 1934. xi+54 pp.

Propriétés des Espaces Abstraits les plus Généraux. Compacité, séparabilité, Transformations et Fonctionelles. By Antoine Appert. Paris, Hermann et Cie, 1934. 55 pp.

These books are numbers 144–146 of the Actualités Scientifiques et Industrielles and form the first three volumes of a series, edited by M. Fréchet, with the title: *Exposés d'Analyse Générale.*

The first volume is intended to give a simple introduction to that part of the theory of transfinite cardinal and ordinal numbers necessary for the understanding of the later volumes. As a consequence, the Zermelo postulate is ignored, the operations of addition and multiplication of transfinite numbers are not defined, and the discussion is restricted to the cardinal numbers $\aleph_0$ and c and to the ordinal numbers of the first and second classes. The latter are defined as the ordinal numbers of well-ordered sets of increasing rational numbers. The treatment is intuitive and few proofs are given. There is a short chapter on operations on sets as well as one on measure and integration, the treatment in the latter being that of W. H. Young. The list of references is short but adequate.

The second volume develops the concepts listed in its sub-title for the very general case of spaces in which the operation of derivation can be defined by means of neighborhoods. Particular attention is paid to the case in which the neighborhoods can be chosen as open sets and a chapter is devoted to the comparison of this condition with the requirement that the derived sets be closed. This, and a chapter on the distributive property of the operation of derivation, completes the discussion of the properties usually thought of as postulates. The various separation postulates are not needed in the development and are scarcely mentioned. There is a short chapter on self-dense and clairsemé sets and a much longer one on connectedness.

The third volume is a continuation of the second. In the first of three chapters various degrees of compactness are defined and the corresponding covering- and product-theorems are proved. In the second chapter a natural extension of the notions of separability and perfect separability is made which leads to generalizations of the theorems of Lindelöf and Cantor-Bendixson. The final chapter deals with continuous transformations and, in particular, with functionals. Semi-continuity of functionals is defined and the relation between the boundedness of functionals having this property and the compactness of their domain of definition is discussed.

These books should form an excellent introduction to the theory of abstract spaces. Dr. Appert has done a good job of exposition, the only "fault" found by the reviewer being that the proofs are frequently given in too much detail. There are a number of interesting examples.

H. E. VAUGHAN

The Quantum Theory of Radiation. By W. Heitler. Oxford, Clarendon Press, 1936. xi+252 pp.

This is the eleventh in The International Series of Monographs on Physics which is published under the editorship of R. H. Fowler and P. Kapitza and it may be fairly said to maintain the very high standard set by the preceding numbers of the series. The first chapter, about one-fifth of the book, is devoted to the classical theory of radiation (Maxwell's equations, electromagnetic potential, field of a moving electron, scattering and absorption of light) and ends with the Hamiltonian form of treatment which is based on the wave equation governing the vector potential and a consequent expansion of the vector potential in a Fourier series (after the time variable has been separated from the space variables). The second chapter treats the quantum theory of the field in free space (the field being regarded as the result of a superposition of plane waves and each of the plane waves having the same Hamiltonian as a linear oscillator). The third chapter discusses the interaction of radiation with matter (emission and absorption, breadth of spectrum lines, photo-electric effect, dispersion and Raman effect, resonance fluorescence, Compton effect). Chapter 4 is devoted to the positive electron and the book closes with a discussion of the penetrating power of high-energy radiation and a summary account of the Born-Infeld non-linear theory of the electromagnetic field.

It may well be imagined from the indication given above of the variety of topics treated that the reading of the book under review is not easy. The boldness of speculation and imagination which is so characteristic of the physical theories of the past decade is continually in evidence and a mathematician, who cannot ride with a carefree heart in the vanguard of attack but must have an ever present anxiety about his lines of communication, may have his moments of discouragement. For instance, the author introduces (p. 71) as a "relativistic analogue of Dirac's well-known δ function" the function

$$\Delta = \lim_{\kappa\to\infty} \frac{1}{\pi r}\left[\frac{\sin \kappa(r+ct)}{r+ct} - \frac{\sin \kappa(r-ct)}{r-ct}\right]$$

and an old-fashioned conscience is worried by a limit which does not exist. But the whole work carries the stamp of authority and one may surmise that these twinges of conscience are caused by nothing more important than matters of elegance which, in the words of Boltzmann, may safely be left to the tailor and the cobbler.

F. D. MURNAGHAN

Les Involutions Cycliques Appartenant à une Surface Algébrique. By Lucien Godeaux. (Actualités Scientifiques et Industrielles, No. 270.) Paris, Hermann, 1935. 44 pp.

The same scheme is used in the present pamphlet as in the preceding ones, namely, to present a brief and fairly popular account of the status of a problem, but accurate and including a suggestion for the direction of most probable further development. Proofs are not given, but the bibliography of fifty-eight titles is cited as occasion arises. This is fairly full, but a number of important recent contributions are not included.

The problem is very clearly stated; it is the study of rational or irrational $(1, p)$ correspondences between the points of two algebraic surfaces, the p points being the successive images of a given one in a birational correspondence of period p. A projective model of each surface is constructed in hyperspace, such that the birational correspondence becomes a collineation. Involutions are then classified according to the number of invariant points. An isolated invariant point may be perfect or imperfect, according as every direction through it does or does not remain fixed. In the case of imperfect points the two invariant directions are examined further. This process is continued until the form of contact at each point is completely accounted for. The procedure is illustrated by a detailed discussion of a plane cyclic collineation.

A regular surface can not have irregular involutions, but the converse is not true. Each combination is discussed, and the criterion obtained in order that a surface shall represent an involution. Finally the theory is applied to surfaces having a canonical curve of order zero.

The booklet is excellently printed on stiff paper, making an attractive page. It furnishes a welcome resumé of this interesting theory.

VIRGIL SNYDER

Gesammelte Werke. By David Hilbert. Volume 3. *Analysis, Grundlagen der Mathematik, Physik, Verschiedenes. Nebst einer Lebensgeschichte.* Berlin, Springer, 1935. 435 pp.

This third volume completes the edition of Hilbert's collected mathematical papers. The last volume spans over a wide range and reminds one again of Hilbert's outstanding contributions to our mathematical knowledge. It contains the papers on Dirichlet's principle, on the calculus of variations, on Hilbert space, and all his papers on physical problems. The series of investigations on the foundation of mathematics has also been reserved for this volume.

One rereads with pleasure Hilbert's famous talk on mathematical problems at the Paris congress in 1900 and one cannot avoid realizing the tremendous strides of the mathematical sciences in the past third of a century. A majority of his problems have been solved as precisely as they were formulated, and for almost all of them one can say that important contributions have been made. One should not forget to mention in this last volume the necrologues on Weierstrass, Darboux, and Hilbert's long-time friends and associates, Minkowski and Hurwitz. They reveal to us more than anything else Hilbert's human qualities and his unusual ability as a writer.

It should be mentioned that some of Hilbert's papers are not to be found in this edition, most of them papers which have been published in similar versions in several periodicals. His books have also been omitted. Through this expedient it was possible to reduce the number of volumes to three instead of the four originally scheduled. One finds, however, a list of all publications, lectures, and dissertations completed under his supervision.

Hellinger has contributed an account of Hilbert's work on integral equations, and Bernays gives an appreciation of his work on the foundation of mathematics. They are both excellent, clearly and well written, but one has a feeling, particularly in regard to the papers on the foundation of mathematics,

that it is somewhat too early to give a full historical evaluation. Blumenthal writes a biography of Hilbert which is rich in details and pleasant reading, and at the same time preserves many facts which are of interest to record.

ØYSTEIN ORE

Psychometrics. By Johnson O'Connor. Cambridge, Harvard University Press, 1934. 291 pp.

The preface to the book states that it is "a study in the human substance of industrial relationships, prosecuted under a grant from the Jacob Wertheim Fellowship on Industrial Relations of Harvard University." The book is written primarily from the point of view of selecting executives by means of tests. The subject matter is essentially highly technical, but it would seem as if the author had expected the book to be read by people of little technical training. It is questionable whether the author has succeeded; if the reader is sufficiently well acquainted with the field to recognize the defects and the merits of the book, he will be irritated by the failure to use standard terminology and by the lack of reference to previous and contemporary work in the same field; if the reader is unacquainted with the subject matter, he will only with difficulty discover the subtleties and premises upon which the work is based,—one of these has already been recognized and discussed publicly.* The book would probably be of little interest to most mathematicians, except those who are interested in the applications of mathematics to the techniques of examining.

C. R. BROLYER

Lehrbuch der darstellenden Geometrie. By E. Müller. Vierte Auflage in drei Teilen. Completely revised by E. Kruppa. Leipzig and Berlin, Teubner, 1936. vi+vi+vi+390 pp.

This well known text now appears in the fourth edition in a completely revised edition by Professor Kruppa of the Technical High School of Vienna. The first part deals with projections on one plane. It is very commendable that, in mapping surfaces and curves, loose expressions like *infinitely close* and *consecutive* are replaced by proper analytic limiting processes. In the second part we find representations of curved surfaces and the solution of problems connected with them. Axonometry, perspective, and geographical mapping form the contents of the third part. Considered as a whole, the greatly abridged fourth edition of this treatise in its modernized form is a distinct improvement over the previous editions and may be warmly recommended to students of descriptive geometry.

ARNOLD EMCH

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* H. S. Conrad, *A note on Johnson O'Connor's formula for the standard error of measurement*, Journal of Applied Psychology, vol. 19 (1935), pp. 725–729.

NOTES

In addition to the members of the editorial staff of the Bulletin, the following persons have assisted the editors, either by refereeing papers or by advising concerning papers offered for publication in the present volume: M. A. Basoco, E. T. Bell, A. A. Bennett, B. A. Bernstein, H. R. Brahana, W. C. Brenke, A. B. Brown, Leonard Carlitz, R. D. Carmichael, E. W. Chittenden, A. B. Coble, H. B. Curry, Arnold Emch, H. T. Engstrom, G. C. Evans, L. R. Ford, Philip Franklin, T. C. Fry, Einar Hille, T. R. Hollcroft, E. V. Huntington, W. A. Hurwitz, M. H. Ingraham, Dunham Jackson, R. D. James, A. J. Kempner, R. E. Langer, D. H. Lehmer, N. H. McCoy, C. C. MacDuffee, W. A. Manning, A. D. Michal, H. H. Mitchell, C. N. Moore, Marston Morse, F. D. Murnaghan, G. Y. Rainich, J. F. Ritt, L. L. Silverman, P. A. Smith, Virgil Snyder, D. J. Struik, J. L. Synge, J. D. Tamarkin, J. M. Thomas, T. Y. Thomas, Stepan Timoshenko, W. J. Trjitzinsky, H. S. Vandiver, J. L. Walsh, Morgan Ward, Louis Weisner, H. S. White, G. T. Whyburn, W. M. Whyburn, Norbert Wiener, R. L. Wilder, S. S. Wilks, John Williamson, W. A. Wilson, B. C. Wong. The editors desire publicly to acknowledge this service.

Harvard University appoints annually two Benjamin Peirce Instructors in Mathematics. These instructorships are intended to give promising young men who already have their doctorates the opportunity to combine their research activities with a moderate amount of teaching, including some advanced instruction. Applications should be sent to Professor W. C. Graustein, Chairman of the Division of Mathematics, by February 15, 1937.

The School of Mathematics of the Institute for Advanced Study each year allocates a small number of stipends to gifted young mathematicians and mathematical physicists to enable them to study and to do research work at Princeton. Candidates must have given evidence of ability in research comparable at least with that expected for the degree of Doctor of Philosophy. Blanks for application may be obtained from the Institute, and are returnable by February 1, 1937.

The National Institute of Sciences of Calcutta, India, plans to issue each year a publication under the title Indian Science Abstracts, the subtitle being An Annotated Bibliography of Science in India. In this publication, the first part of which appeared recently, it is planned to include abstracts of all scientific papers published in India, and also of papers published abroad on work done in India or based on Indian material.

The Anniversary Meeting of the Founder Societies of the American Institute of Physics was held in New York City, October 29–31.

An Indian Science Congress will be held at Hyderabad on January 2–8, 1937. There will be a section for mathematics and physics.

The Thomas Hawksley Lecture of the Institution of Mechanical Engineers

was delivered on November 6 by Professor A. Fowler. His subject was *The spectroscope and the atom.*

The note on p. 620 of the present volume of this Bulletin regarding the award of the Fields Medals of the International Congress of Mathematicians at Oslo is in error. It was not Professor K. A. M. Ahlfors, of the University of Helsingfors, who received the medal, but Professor L. V. Ahlfors, of Harvard University.

The University of Paris conferred the honorary doctorate upon Dean G. D. Birkhoff, of Harvard University, on August 12, 1936.

Professor Max Planck, president of the Kaiser Wilhelm Society for the Advancement of Science, has been named doctor honoris causa by the Faculty of Philosophy of the University of Graz.

Dr. N. S. Osborne, of the United States Bureau of Standards, received an honorary doctoral degree from the Michigan College of Mining and Technology for his researches in the properties of ammonia and steam.

Dr. H. L. Garabedian has been promoted to an assistant professorship at Northwestern University.

The retirement of Professor Anna D. Lewis, of Lake Erie College, is announced.

Assistant Professor L. H. McFarlan, of the University of Washington, has been promoted to an associate professorship in mathematics.

Acting Dean W. J. Miller, of the University of North Carolina, has been appointed professor of electrical engineering at the University of Alabama.

Assistant Professor A. F. Moursund, of the University of Oregon, has been promoted to an associate professorship.

Dr. Rufus Oldenburger has been promoted to an assistant professorship at the Armour Institute of Technology.

Dr. W. T. Reid, of the University of Chicago, is spending the first part of this academic year at the Institute for Advanced Study.

Assistant Professor I. S. Sokolnikoff, of the University of Wisconsin, has been promoted to an associate professorship.

Dr. Ruth W. Stokes, of Mitchell College, has been appointed professor of mathematics at Winthrop College.

Dr. Otto Szász has been appointed research lecturer at the University of Cincinnati for the academic year.

Dr. Max Born, of the University of Cambridge, has been appointed to a professorship in natural philosophy at the University of Edinburgh.

Dr. Karl Grünholz, has been appointed professor of mathematics at the University of Würzburg.

Dr. H. G. Hooker, of Christ's College, has been appointed assistant lecturer in mathematics at the University of Cambridge.

Dr. J. H. C. Thompson has been elected to a fellowship and lectureship in mathematics at Wadham College, University of Oxford.

Professor L. V. Ahlfors, recently of the University of Helsingfors, and now of Harvard University, has been elected a member of the Societas Scientiarum Fennica.

Dr. H. M. Bacon, of Stanford University, has been promoted to an assistant professorship.

Dr. A. H. Bailey, of Ohio State University, has been appointed to a professorship in mathematics at Central Normal College, Danville, Indiana.

Assistant Professor W. D. Baten, of the University of Michigan, has been appointed associate professor of mathematical statistics and research associate in statistics at Michigan State College. Professor Baten will devote a portion of his time to teaching and research in the department of mathematics and the remainder as statistical adviser in the Michigan State College Experiment Station.

Major A. P. Cowgill has been appointed to a professorship in mathematics at the Indiana Technical College, Fort Wayne.

Dr. J. L. Dorroh has been appointed to an associate professorship in mathematics at Judson College, Marion, Ala.

Dr. G. R. Trott, of Johns Hopkins University, has been appointed to a professorship at Blue Mountain College, Miss.

Mr. W. A. Vezeau has been appointed to a professorship at St. Joseph's College, Philadelphia, Pa.

Assistant Professor H. S. Wall has been promoted to an associate professorship at Northwestern University.

Assistant Professor Louis Weisner of Hunter College (of the City of New York) has been promoted to an associate professorship.

Professor E. P. Wigner, of Princeton University, has been appointed to a professorship in physics at the University of Wisconsin. The appointment begins February 1, 1937.

The following appointments to instructorships are announced: Armour Institute of Technology: Dr. G. C. Webber; Brown University: Dr. G. B. Price, Mr. Robert Rawhouser; Butler University: Dr. B. C. Getchell; Columbia University: Dr. F. J. Murray; Cornell University: Dr. L. L. Lowenstein, Dr. C. E. Rhodes; Georgia School of Technology: Mr. W. B. Coleman; Iowa State College: Dr. R. A. Higdon; Michigan State College: Mr. J. D. Hill, Dr. C. C. Hurd; New York University: Dr. Leo Zippin; Northwestern University: Dr. W. C. Randels; Notre Dame University: Rev. J. H. Kenna; Princeton University: Mr. Harwood Rosser (part time); South Dakota State College: Dr. L. E.

Mehlenbacher; Southern Methodist University: Mr. L. D. Rodabaugh; Trinity College: Mr. J. F. Wyckoff; Tulane University: Dr. E. G. H. Comfort; Whitman College: Mr. I. E. Highberg.

Among members of the American Mathematical Society who have entered the field of secondary teaching are the following: Dr. R. C. Stauffer, at Miss Fine's School, Princeton, N. J.; Dr. P. L. Trump, at Wisconsin University High School, Madison, Wis.

Emeritus Professor E. J. Nanson, of the University of Melbourne, Australia, died on July 1, 1936, at the age of eighty-five years.

Professor Salvatore Pincherle, of Bologna, died on July 10, 1936, at the age of eighty-three years.

Dr. R. R. Webb, of St. John's College, Cambridge, died on July 29, 1936, at the age of eighty-six years.

Professor Charles Hamilton Ashton, a member of this Society since 1901, died on August 2, 1936, at the home of his daughter, Mrs. Leon Becker, in Seattle, Washington. He was born in 1866 in Cambridge Valley in New York. He held degrees from Union College (A.B., 1887; Sc.D., 1922), from Harvard University (A.M., 1894), and from the University of Munich (Ph.D., 1909). His chief field of study was the theory of functions of complex variables. He taught for eight years at Harvard University, and for thirty-three years at the University of Kansas, where he was chairman of the department from 1917 to 1931. He continued his work as professor of mathematics until last June.

Dr. Julia Trueman Colpitts, Associate Professor of Mathematics at Iowa State College, and a member of this Society since 1907, died on August 8, 1936, at Southampton, England, as she was returning from the International Congress at Oslo. Miss Colpitts was born on February 22, 1875, at Point de Bute, N.B. She held degrees from Mount Allison University (A.B.) and from Cornell University (A.M. and Ph.D.), her special field being the theory of functions. From 1900 until the time of her death, she belonged to the faculty of Iowa State College. Recently she had been chairman of the Iowa Section of the Mathematical Association of America and the national president of Sigma Delta Epsilon.

Dr. H. B. Hedrick, of Washington, D. C., died on October 7 at the age of seventy-one years. Until his retirement in 1932 he was chief ballistician at the Aberdeen Proving Grounds. Previously he was astronomer at the U. S. Naval Observatory and at Yale University and mathematician of the department of terrestrial magnetism of the Carnegie Institution of Washington.

The death of Mr. H. H. Lansford, of Brooklyn College, is announced.

Professor E. B. Lytle, of the University of Illinois, died on September 5 at the age of sixty-one years. He had been a member of the Society since 1904.

Professor A. A. Titsworth, of Rutgers University, died on August 15, at the age of eighty-three years. He had been professor of mathematics in that institution for forty-two years.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

391. Professor E. P. Lane and Professor M. L. MacQueen: *Asymptotic curves on a surface.*

The purpose of this paper is to make some contributions to the theory of the asymptotic curves on an analytic non-ruled surface in ordinary projective space. The coordinates of a variable point on the surface are regarded as solutions of a system of linear homogeneous partial differential equations which are written in Fubini's canonical form. Power series expansions in non-homogeneous projective coordinates for the parametric asymptotic curves on the surface are computed to terms of higher degree than have hitherto been considered. By means of these power series a study is made of the asymptotic curves regarded as space curves. Some progress is made in this paper toward the solution of the problem of finding all surfaces on which the asymptotic curves are twisted cubics. Some properties of such surfaces are presented. (Received September 1, 1936.)

392. Professor L. M. Blumenthal: *A note on horn angles and abstract metrics.*

Kasner has shown that attached to an ordered pair C_1, C_2 of analytic curves passing through a point P in the same direction, is the unique (regular) conformal invariant $I_{12} \equiv (d\gamma_2/ds_2 - d\gamma_1/ds_1)/(\gamma_2-\gamma_1)^2$, $(\gamma_1 \neq \gamma_2)$; the γ's are the curvatures of C_1, and C_2, respectively, at P. A recent paper by Kasner and Comenetz (Proceedings of the National Academy of Sciences, vol. 22 (1936), pp. 303–309) derives "a fundamental identity in conformal plane geometry, relating the conformal invariants of adjacent horn angles." An inequality satisfied by the three conformal invariants of three horn angles formed by three curves at a point is also obtained. In this note an abstract space K is considered whose points are ordered pairs of real numbers, with the distance P_1P_2 of the points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ defined as $P_1P_2 = (y_2-y_1)/(x_2-x_1)^2$, $x_2 \neq x_1$. The group having the distance defined above as the unique invariant of two points is $x' = ax+b$, $y' = a^2y+c$. The fundamental identity found by Kasner and Comenetz is merely the four-points relation for the space K, and is easily exhibited in determinantal form, while the triangle inequality for three horn angles is, in this interpretation, immediate. Many properties of the space K

give rise to new problems in the conformal geometry of horn angles. (Received September 24, 1936.)

393. Professor L. M. Blumenthal: *New metric foundations of hyperbolic geometry.*

New necessary and sufficient conditions (in terms of distance relations of certain *quadruples* of points) are found that to each element of a given semi-metric space M an ordered set of n real numbers may be attached, in such a way that if x and y are any two elements of M, and $(x_1, x_2, \cdots, x_n)$, $(y_1, y_2, \cdots, y_n)$ are the n-tuples attached to x and y respectively, then the hyperbolic distance function applied to the two n-tuples shall determine a number equal to the previously defined distance xy. (Received October 8, 1936.)

394. Professor T. R. Hollcroft: *The web of quadrics.*

The characteristics of a web of quadrics, with and without basis elements, have never been completely or accurately given. This paper gives a brief summary of the results obtained by many mathematicians during the past seventy-five years, correcting errors and completing the sets of characteristics of both the web of quadrics and its associated involution. The characteristics of a web of quadric cones (a web of quadrics with a basis double point) are derived. The nature of these characteristics is entirely different from that of a web of non-singular quadrics. (Received September 1, 1936.)

395. Dr. Reinhold Baer: *Primary abelian groups and their automorphisms.*

To every group A of (proper) automorphisms of the primary abelian group G corresponds the subgroup $F(A)$ of its fixed elements in G and the subgroup $C(A)$, generated by the commutators $x-xf$ with x in G and f in A. Those subgroups of G which are groups $F(A)$ and those which are groups $C(A)$ for a suitable group A of automorphisms of G are characterized by their structure and by their situation in G. The centralizer of the group of all the automorphisms of G appears to be essentially the group of those automorphisms of G which map every subgroup of G upon itself and this allows one to characterize those primary abelian groups whose group of automorphisms is abelian. (Received September 23, 1936.)

396. Dr. Reinhold Baer: *Dualism in abelian groups.*

A dualism d in an abelian group G is a one-one correspondence between the subgroups of G such that $S \leqq T$ if, and only if, $Td \leqq Sd$ and such that S and $G/(Sd)$ are isomorphic. The existence of a dualism in a finite abelian group has been noticed by E. Steinitz. It is remarkable that some sort of converse holds true. For there exists a dualism in the abelian group G if, and only if, G is a group without elements of infinite order whose primary components are finite. If there exists a dualism, then there exists even a dualism d which satisfies: A/S and Sd are isomorphic; there exists an automorphism of G, mapping S upon T, if and only if there exists an automorphism of G, mapping Sd upon Td. (Received September 10, 1936.)

397. Dr. Reinhold Baer: *Equivalence of algebraic extensions.*

If A and B are equivalent algebraic extensions of their common subfield C, then the same equations in C have solutions in A and in B. The converse holds true if, for example, A is separable with regard to C. If A and B are not separable with regard to C, then it is possible to prove an analogous proposition under somewhat weaker assumptions. (Received September 10, 1936.)

398. Dr. George Comenetz: *Conformal geometry on a surface.*

This paper generalizes to the case of curved surfaces certain theorems on plane conformal geometry due to Kasner (Proceedings of the Fifth International Congress of Mathematicians, Cambridge, vol. 2 (1912), p. 81; Proceedings of the National Academy of Sciences, vol. 22 (1936), p. 303; and unpublished work). In this paper are considered properties of figures on a surface which remain invariant when the surface is transformed conformally into any other surface. Let two curves be tangent at a point O, and let Γ, G be their respective geodesic curvatures at O. Let Γ_1, G_1 denote the values of the first derivatives of curvature with respect to arc-length at O; Γ_2, G_2 the second derivatives, $\cdots$. Then if the curves have first order contact, the quantity $(G_1-\Gamma_1)/(G-\Gamma)^2$ is an invariant, just as in the plane case. But for second order contact $(G=\Gamma)$, the invariant found is $[4(G_1-\Gamma_1)(G_2-\Gamma_2)-5(G_2-\Gamma_2)^2-4(\Gamma^2+K)(G_1-\Gamma_1)^2]/[G_1-\Gamma_1]^3$, where K is the Gaussian curvature of the surface at O. For a curvilinear right angle, the new invariant differs from the known invariant in the plane case by the addition of $-K(G_1-\Gamma_1)+2(GK_s-\Gamma K_\sigma)+(-gK_s+\gamma K_\sigma+K_{s\sigma}+K_{\sigma s})/2$ to the numerator, where the subscripts σ, s indicate differentiation with respect to arc-length along the curves of an arbitrary orthogonal net tangent to the right angle at O, and γ, g are the curvatures of the curves of the net at O. (Received September 19, 1936.)

399. Dr. George Comenetz: *Schwarzian reflection on a surface.*

Let C be a given curve on a surface S. The Schwarzian reflection, or *conformal symmetry*, determined by C may be defined as the anti-conformal transformation of S into itself which leaves the points of C fixed. Take any curve D on S intersecting C at a point O, and let $\overline{D}$ be the image of D under the symmetry. Let γ be the geodesic curvature of C at O; γ_1 the first derivative of curvature with respect to arc-length, at O; γ_2 the second derivative; $\cdots$. Let Γ, Γ_1, Γ_2, $\cdots$, be the corresponding quantities for D at O; and $\overline{\Gamma}$, $\overline{\Gamma}_1$, $\overline{\Gamma}_2$, $\cdots$, for $\overline{D}$ at O. Call the angle from C to D at O, θ. Then $\Gamma+\overline{\Gamma}=2\gamma\cos\theta$, and $\Gamma_1+\overline{\Gamma}_1=2\gamma_1\cos 2\theta$, just as when S is a plane. But $\Gamma_2+\overline{\Gamma}_2=2\gamma_2\cos 3\theta-2\gamma_1(\Gamma-\overline{\Gamma})\sin 2\theta+2\gamma(\Gamma_1-\overline{\Gamma}_1)\sin\theta+K_s\sin\theta\sin 2\theta$, where K_s is the derivative at O of the Gaussian curvature of S, with respect to arc-length perpendicular to C. The expression for $\Gamma_3+\overline{\Gamma}_3$ is obtained when $\theta=0°$ and $\theta=90°$, and is applied to calculate the conformal invariant of a right angle from the invariant of a horn angle of second order contact. When S is a plane, the results become those obtained by Kasner (references in previous abstract). (Received September 19, 1936.)

400. Dr. A. D. Fialkow (National Research Fellow): *Initial motion at a point of equilibrium.*

A plane positional field of force gives rise to the three-parameter family of dynamical trajectories. A differential-geometric characterization of this system of curves at regular points of the field has been given by Kasner. The author considers similar questions in the neighborhood of an isolated point of equilibrium. The following theorem is analogous to the first of Kasner's set of characteristic properties: The cubics osculating each of the ∞^1 trajectories passing through a point of equilibrium in the same direction at the given point have asymptotes which form a pencil whose center lies on the initial tangent. The qualitative nature of the lines of force near the point of equilibrium has been studied by Poincaré, Bendixson, and others. By means of some of their results, a few elementary connections between the differential geometry of the paths and the general character of the lines of force are found. (September 24, 1936.)

401. Dr. R. A. Harrison: *Cremona webs of surfaces in S_3 without base curves.*

Cremona transformations $T_{n-n'}$, in three-way space S_3, in which n' has the maximum value n^2, have no fundamental curves for the web of surfaces (ϕ_n) of order n. The surfaces of the web have a common $(n-1)$-fold point; and, if the tangent cone at this point, the vertex, is not fixed, the transformations are necessarily of order 2, 3, or 4. After consideration of these transformations, the more general case, in which the tangent cone at the vertex is fixed for the system, is taken up. Finally, T_{2-4} is extended to T_{2-3} in S_4 and generalized to $T_{2-2^{(n-1)}}$ in S_n, while the possible generalization of T_{3-9} and T_{4-16} is noted. (Received October 5, 1936.)

402. Professor Edward Kasner: *Conformal theorems on triplets of curves.*

A triplet T is defined as three curves $C_1C_2C_3$ passing through a common point in a common direction. The three horn angles determine three conformal invariants I_{12}, I_{23}, I_{13}, or three natural measures M_{12}, M_{23}, M_{13} (see Proceedings of the National Academy of Sciences, April, 1936), for which a new type of triangular inequality is valid. In the equality case $M_{12}+M_{23}=M_{13}$ the triplet is called wide-open (or linear). For the general triplet C_3 may be replaced by C_3' in such a way that M_{12} and M_{23} are the same and the new triplet $C_1C_2C_3'$ is linear. Two triplets whose three corresponding angles have equal measures are not necessarily conformally equivalent. Thus $C_1C_2C_3$ may be replaced by $C_1C_2C_3^*$. The curves C_3 and C_3^* are then said to be "linear symmetric" with respect to the linear system determined by C_1C_2. If the three horn angles in T are bisected (by Schwarzian or conformal symmetry), the new triplet has measures half of the old. When bisection is iterated indefinitely a limiting curve C_0 is obtained which may be said, in a new conformal sense, to be the trisection point of the three median linear systems. (Received September 26, 1936.)

403. Mr. A. P. Morse: *Convergence in variation and related topics.*

In view of a result due to Adams and Clarkson (this Bulletin, vol. 40 (1934), pp. 413–417, Theorem 6) it is natural to raise the general question: what transformations of a sequence will preserve convergence in variation? Phrasing the problem in terms of complex functions of a real variable, it is proposed in the present paper to answer this question as fully as possible. An incidental result is a theorem on change of variable in a Lebesgue integral, generalizing earlier theorems of de la Vallée Poussin and Fichtenholz. Some applications of the main theorem are: convergence in length (see Adams and Lewy, Duke Mathematical Journal, vol. 1 (1935), pp. 19–26) is an additive and multiplicative property of sequences whenever *one* of the limit functions is absolutely continuous (extending Theorem 6 of Adams and Lewy); a necessary and sufficient condition for convergence in length is convergence in variation plus either convergence in measure or "almost convergence in the mean" of the derivatives (the latter type of convergence is intermediate between convergence in measure and convergence in the mean). The paper concludes with a necessary and sufficient condition for convergence in the mean and several generalizations of a theorem of Plessner (Journal für Mathematik, vol. 160 (1929), pp. 26–32, Theorem 1). (Received September 26, 1936.)

404. Dr. F. J. Murray (National Research Fellow) and Professor John von Neumann: *On rings of operators.* II.

This paper is a continuation of one by the same authors, *On rings of operators*, Annals of Mathematics, vol. 37 (1936), pp. 116–229. The authors investigate further the properties of factor rings in case II. It is shown that there exist elements $g_1, \cdots, g_m$ such that the trace $Tr_M(A)$ is expressible as $\sum_{i=1}^{m} (Ag_i, g_i)$. This implies the linearity and weak continuity of the trace. If M' is also in case II_1 and in the standard normalization $C=1$ (cf. loc. cit., Theorem X), then m may be taken as 1 and M, M' and a subset $\mathfrak{A}$ of $\mathfrak{H}$ are isomorphic. It is possible to complete M (and M') by adding operators A which are ηM (or $\eta M'$) so that the resulting set is isomorphic to $\mathfrak{H}$. Finally it is shown that if M_1 and M_2 are factors satisfying these conditions then an algebraic isomorphism implies an inner spatial isomorphism between them. (Received September 3, 1936.)

405. Dr. G. B. Price: *On the extreme points of convex sets.*

This paper investigates the properties of extreme points of convex sets in spaces which are linear and normed. If the metric has the property that the segment joining any two points of the spherical neighborhood $S(x, r)$ is interior to the neighborhood except at most for the two points themselves, the space is denoted by L^*, and in all other cases by L. The study of extreme points is far simpler in spaces L^* than in spaces L, and the results are more complete. The results obtained include the following: An approximation theorem first proved by Minkowski for euclidean 3-space is generalized in two ways for spaces L and L^*. If a set G in a Banach space can be approximated in a specified manner by compact sets, then G itself is compact. These two results lead to a new proof

that the convex hull of a compact set in a Banach space is compact. The closed convex hull of the set of extreme points of a closed, compact, convex set in a space L_p is the convex set itself. Furthermore, the relation between a set S and the extreme points of its closed convex hull and also the extreme points of the limit set of a sequence of closed, compact, convex sets are studied in detail. (Received September 9, 1936.)

406. Dr. W. T. Reid: *On the Jacobi condition for the double integral problem of the calculus of variations.*

This paper is concerned with the Jacobi condition for the problem of minimizing $I=\iint_A f(x, y, z, z_x, z_y)dxdy$ in a class of admissible surfaces $z=z(x, y)$ with fixed values on the boundary C of A. The usual form of the Jacobi condition states that along a non-singular minimizing extremal surface E there can exist no solution $u(x, y)$ of the accessory equation defined in a neighborhood of a simply closed regular curve C_0 and its interior A_0 in A, with $u\equiv 0$ on C_0 and $u\not\equiv 0$ in A_0. It is proved in the present paper that the above condition is still true when the phrase "simply closed regular curve C_0 and its interior A_0" is replaced by "connected open set $\mathcal{A}$ and its frontier $\mathcal{C}$," "A_0" by "$\mathcal{A}$" and "C_0" by "$\mathcal{C}$." When E is not an extremal, but of class C', a corresponding theorem is proved in terms of the Haar form of the accessory equations. The formulation of the Jacobi condition in this latter case, however, involves the additional assumption $|u_x|+|u_y|\not\equiv 0$ on $\mathcal{C}$. The extended form of the Jacobi condition here proved is of use in the study of the associated accessory boundary problem. The principal results of this paper were obtained in the summer of 1933, on the occasion of Professor Bliss' lectures at the University of Chicago on multiple integral problems of the calculus of variations. (Received October 3, 1936.)

407. Mr. L. B. Robinson: *A determinant used in functional equations.*

Consider the determinants of infinite order $\Delta^{(n-1)}\equiv|\delta_{ij}+a_{ij}|$, where $\delta_{ii}=1$, $\delta_{i+1,i}=S_1+S_2+S_3+\cdots+S_{n-1}$, $\delta_{i+2,i}=S_1S_2+S_1S_3+\cdots+S_{n-2}S_{n-1}$, $\cdots$, $\delta_{i+(n-1),i}=S_1S_2\cdots S_{n-1}$, $|S|<1$. The other δ's vanish. Extract from the above the upper left-hand determinant of $p+r$ and p rows respectively, that is, $\Delta_{p+r}^{(n-1)}$, $\Delta_p^{(n-1)}$. Write the inequality $|\Delta_{p+r}^{(n-1)}-\Delta_p^{(n-1)}|<\epsilon$, $r=1, 2, 3, \cdots$, where ϵ is as small as one pleases and n is greater than some arbitrary number. If this inequality be always satisfied then $\Delta^{(n-1)}$ converges. It can be proved by addition and subtraction of rows that $\Delta_{p+r}^{(n-1)}\equiv\Delta_{p+r}^{(n-2)}\equiv\cdots\equiv\Delta_{p+r}^{(0)}$ the last being the normal determinant of Poincaré. The same is true of Δ_p. But $|\Delta_{p+r}^{(0)}-\Delta_p^{(0)}|<\epsilon$. The author has proved the convergence of $\Delta^{(n-1)}$. (Received September 29, 1936.)

408. Dr. A. R. Schweitzer: *On the definition of "sense" in the foundations of geometry.*

This paper is in continuation of *A theory of geometrical relations*, American Journal of Mathematics, 1909, 1913. Dr. Schweitzer gives four definitions of "sense" in the foundations of geometry and discusses their mutual interrela-

tions. The first definition is an n-dimensional generalization ($n=1, 2, 3, \cdots$) of the definition given by the author in the Transactions of this Society, 1909, p. 313. The second is an application of the abstract definition given by the author in the American Journal of Mathematics, 1909, p. 372. The third is phrased in terms of the author's relations B_n and S_n, as indicated in the American Journal of Mathematics, 1909, p. 366. The fourth definition is topological and is based on the author's relation I_n. These four definitions correspond roughly to standpoints in the theory of definitions of "sense" of (1) Pieri, (2) Hilbert, (3) Thaer, Jahresbericht der Deutschen Mathematiker Vereinigung, vol. 28 (1919), and (4) Klein, Veblen, and J. R. Kline. (Received September 8, 1936.)

409. Dr. S. E. Warschawski: *On the behavior of a conformal mapping at a cusp.* II.

The present paper is a continuation of the preliminary report given in abstract 42-5-219. Let C be a closed Jordan curve in the w-plane with a cusp at $w=0$ which is characterized by a function $\phi(\rho)$ as described in that abstract. The following are the results obtained in this paper: 1. Let $z(w)$ map the interior R of C conformally on $|z+1|<1$ and let $z(0)=0$. If there exists an increasing function $\mu(\rho)$ such that $|d\phi(\rho)/d\rho| \leq \mu(\rho)$ and $\int_0^\infty \mu(\rho)^2\rho d\rho/\phi(\rho)$ converges, then, for $|w|=r$, $\lim_{r\to 0}[|z(w)|/\exp(-\pi\int_r^\infty d\rho/\rho\phi(\rho))]=\lambda$ exists in $R+C$ and λ is *positive*. 2. Let $z(w)$, instead of representing R on a circle, map R on another region R' of the z-plane, bounded by a closed Jordan curve C' with a cusp at $z=0$, which is characterized by a function $\psi(\rho)$ defined in the same manner as $\phi(\rho)$. A *necessary and sufficient* condition, involving $\phi(\rho)$ and $\psi(\rho)$ only, is given in order that the "dilatation" of the map at $w=0$, that is, $\lim_{w\to 0}|z(w)|/|w|$, be a *definite finite* number. (Received October 3, 1936.)

410. Professor Hassler Whitney: *On the maps of an n-sphere into another n-sphere.*

An elementary and simple proof is given of the fact that two maps of one n-sphere into another n-sphere are homotopic if and only if they have the same degree. (Received September 26, 1936.)

411. Professor Kurt Friedrichs: *On certain inequalities and characteristic values problems for analytic functions and for functions of two variables.*

In the first part of the paper the author establishes an inequality for $\iint w^2 dxdy$ under the conditions $\iint |w|^2 dxdy < \infty$ and $\iint w dxdy = 0$. This inequality leads to a complete solution of the reduction of the quadratic form $\iint w^2 dxdy$ relative to the form $\iint |w|^2 dxdy$. The rexults of the first part are applied in the second part to the theory of equilibrium and vibration of an elastic plate. (Received October 10, 1936.)

412. Dr. E. L. Post: *Finite combinatory processes. Formulation* 1.

The present formulation envisages a general problem consisting of a class of specific problems. A symbol space is postulated in which each specific prob-

lem and corresponding answer can be represented, and in which the work leading from symbolized problem to symbolized answer is to be carried out. A fixed set of directions both directs operations in the symbol space and determines the order in which those directions are to be applied. It gives a solution of the general problem if on applying it to each specific problem the process thus set up terminates in the answer to that problem. The present formulation describes a specific symbol space, enumerates the primitive processes the problem solver is assumed capable of performing in that symbol space, and gives a definite structure to the set of directions. Its purpose is to present a model for formulations of increasing psychological complexity and a norm for their logical reduction, the whole to yield a theory of the limitations of mathematics along the lines already initiated by Gödel and Church. (Received October 31, 1936.)

413. Professor A. A. Albert: *A note on the matrices defining total real fields.*

A field K of degree n over a field F of real numbers is called total real if its complex conjugate fields over F are all fields of real numbers. The author gives a matric construction of all such fields by proving that K is total real over F if and only if K is equivalent over F to $F(Z)$, $Z=DS$, where D is an n-rowed diagonal matrix with diagonal elements positive numbers of F and S in any symmetric matrix with elements in F for which the characteristic function of DS is irreducible in F. The irreducibility condition is a usual final condition in problems on the construction of fields. (Received November 2, 1936.)

414. Professor A. A. Albert: *Normalized integral bases of algebraic number fields.* I.

A generation $F=R(\theta)$ of an algebraic number field F of degree n over the field R of all rational numbers is called primitive if θ has zero trace and $1, \theta, \omega_3, \cdots, \omega_n$ form a basis of the algebraic integers of F. The author proves that every F has a primitive generation and obtains necessary and sufficient congruential conditions on the minimum function of θ that θ provide a primitive generation. These conditions are applied to cubic, quartic, and quintic fields to obtain explicit coefficient restrictions in these cases. The normalization to a primitive generation is applied to simplify the author's explicit determination of the integers of all cubic fields (Annals of Mathematics, 1930) and subsequently further normalizations are also obtained. It is then shown that every cubic field is equivalent to one of a set of certain classes of fields such that no field in any one class is equivalent to a field in any other. The classification is an invariantive one in terms of the discriminant of the field and certain other invariants, and the results give an explicit determination of the discriminant of every cubic field in terms of the coefficients of the minimum function in a normalized generation. (Received November 2, 1936.)

415. Mr. C. B. Allendoerfer: *The imbedding of Riemann spaces in the large.*

The classical theory of the imbedding of a Riemann space in a euclidean space has recently been revised in an elegant manner by W. Mayer (Trans-

actions of this Society, vol. 38, pp. 267–309). In Mayer's theory the imbedding is defined by a number of forms, but precise conditions which the coefficients of these forms must satisfy are not given. Part I of the present paper obtains these conditions in concise form and recasts the entire theory so that it applies to any simply connected Riemann manifold. In part II there is defined an arithmetic invariant, τ_p, of a Riemann space for every positive integer p. It is shown that if a space with $\tau_p \geqq 3$ can be imbedded such that p is the rank of its second fundamental form, then the imbedding is rigid. Similarly if $\tau_p \geqq 4$, it is shown that the Codazzi equations of the classical theory are algebraic consequences of the Gauss equations. (Received November 6, 1936.)

416. Dr. J. L. Brenner: *The normal subgroups of the groups of matrices mod p^r. II.*

G is the group of non-singular n^2 matrices mod p^r. G is isomorphic with the group of automorphisms of the abelian group of order p^{nr} and type $(r, r, \cdots, r)$. All the normal subgroups of G are found when $p=2$. If $n \neq 2$, the results parallel those when $p>2$ (see abstract 42-7-275). Let $s((a_{ij})) = \min_{i \neq j} (a_{ij}, a_{ii}-a_{jj}, p^r)$. Normal subgroups N are classified according to N', the subgroup of N of determinant 1 (p^r), and N'', the subgroup of N' which satisfies $a_{ii} \equiv 1 (p^r)$, $s = \min s(A)$, $A \in N$. The classification involves four natural parameters, or five if $n=2$. (Received November 2, 1936.)

417. Mr. J. F. Daly: *On the points of an algebraic manifold not reachable by a given parametric representation.*

It is well known that a given parametric representation of an irreducible algebraic manifold will not in general cover the manifold completely. This paper shows, however, that if the base field is the complex field, than any point of the associated manifold not obtainable by an allowable specialization of the given parameters is always a limit point of points which are thus obtainable. The proof consists essentially in choosing a new transcendental basis for the field defining the parametric representation, and then employing a theorem of Ritt. (Received October 29, 1936.)

418. Professor W. L. Duren: *Contractible problems of Bolza.*

Suppose that the functions which define admissible arcs for a non-parametric problem of Bolza in the calculus of variations can be divided into two groups, $y_h(x)$, $(h=1, \cdots, r)$ and $z_s(x)$, $(s=1, \cdots, l)$, $(r+l=n)$, in such a way that the side conditions determine a one-to-one correspondence between the admissible sets $y_h(x)$, $z_s(x)$ and the sets $y_h(x)$. Then the problem is said to be contractible. For the case that the end conditions do not involve the end values of the functions $z_s(x)$ this paper develops conditions which are sufficient to insure that an extremal arc E furnishes a minimum with respect to arcs having points (x, y) neighboring those of E. The fields used are based upon an invariant integral in xy-space and are constructed from families of extremals all of which satisfy the conditions $F_{z'_s}=0$ identically. A Weierstrass condition stronger than the conventional one is required. (Received November 6, 1936.)

419. Professor Arnold Emch: *On cremona involutions connected with the Weddle surface and some covariant surfaces.*

This paper deals with involutorial cubic and Geiser transformations T_3 and T_7 connected with the Weddle surface. It is shown that the product of T_3 and T_7 is a T_9 of order nine with the property $T_9 = T_7T_3 = T_3T_7$. The properties of T_9 lend to a new covariant sextic surface F_6 which is pointwise invariant in T_9. Finally a parametric representation of F_6 and similar surfaces is given. (Received October 28, 1936.)

420. Mr. D. W. Hall and Dr. G. E. Schweigert: *On non-n-alternating transformations.*

A study is made of non-alternating transformations $T(A) = B$ on compact connected sets where the transformation satisfies either of the conditions: (1) if for any point x of B there exists a cutting Σ of $A - T^{-1}(x)$ consisting of at most n points, then there is no point y of B such that the inverse of y intersects both sets of the separation $A - [T^{-1}(x) + \Sigma]$; (2) for any component K of $B - x$, the inverse of K is not separated by the removal of any n of its points. Transformations of the second type are also of the first type on locally connected sets. Let A be a set every two points of which are separated by at most k points. If B is non-degenerate and $k \leqq n$, then T is not of the second type; furthermore T is of the first type if and only if it is homeomorphic; while if $k = n+1$, every T of the first type is A-set reversing (see G. T. Whyburn, American Journal of Mathematics, vol. 58, p. 311) and each T of the second type sends every k cutting of A into a single point. (Received October 30, 1936.)

421. Professor C. G. Latimer: *The classes of integral sets in a quaternion algebra.*

A generalized quaternion algebra $\mathfrak{A}$ contains an infinitude of integral sets. All the sets equivalent to a given set are said to form a class. It is shown that the norm of the general element of trace zero in an integral set $\mathfrak{G}$ is a ternary quadratic form f, which belongs to a class uniquely determined by the class of sets containing $\mathfrak{G}$. Moreover, the genus G which contains f or $2f$, according as the fundamental number d of $\mathfrak{A}$ is even or odd, is uniquely determined by $\mathfrak{A}$. It is shown that there is a one-to-one correspondence between the classes of integral sets in $\mathfrak{A}$ and all the classes in G or all the improperly primitive classes in G, according as d is even or odd. From this and a known result on ternary forms, it follows that if $d < 0$, then any two integral sets in $\mathfrak{A}$ are equivalent. (Received November 5, 1936.)

422. Professor Tibor Radó: *On the harmonic majorants of subharmonic functions.*

Let $u(x, y)$ be subharmonic in a bounded domain (connected open set) G. Denote by D a Dirichlet domain comprised in G together with its boundary. For such domains D one has (F. Riesz, Acta Mathematica, vol. 48, p. 334) a *best harmonic majorant* h for u in D, defined solely in terms of the values of u on the boundary of D. Besides, F. Riesz has also considered a *least harmonic majorant* h^* of u in D, defined in terms of the values of u in D itself (Acta Math-

ematica, vol. 54, pp. 353–358). *The purpose of this paper is to prove that $\bar{h}$ is always equal to h^*.* As far as the author is aware, this was only known for special types of subdomains. The result can be stated in the following equivalent form which seems to express more adequately its true meaning. Denote by h a function which is harmonic in D and define in G a function v as follows: $v=h$ in D and $v=u$ in $G-D$. If this function v is again subharmonic in G, then let us say that h is admissible for u in D. One has then the following theorem. *If D is a Dirichlet domain comprised in G together with its boundary, then there exists exactly one harmonic function which is admissible for u in D.* (Received October 14, 1936.)

423. Dr. J. B. Rosser: *Gödel theorems for non-constructive logics.*

By "Carnap's Rule" shall be meant the rule which can be roughly stated: "If $f(0)$, $f(1)$, $f(2)$, $\cdots$, then (x) $f(x)$." The logics got by starting with the system of Principia plus Peano's axioms and allowing 1, 2, $\cdots$, ω, $\omega+1$, and so on up to, but not including, ω^2 uses of Carnap's Rule (interspersed with uses of the ordinary rules) are studied. It is proved that if one of these logics is simply consistent, then this logic contains undecidable propositions, does not contain the formula stating the simple consistency of the logic, and is not closed under Carnap's Rule. (Received October 20, 1936.)

424. Dr. W. E. Sewell: *A note on the development of functions in a sequence of normal and orthogonal polynomials in the complex domain.*

An application of recent results of the author (Proceedings of the National Academy of Sciences, vol. 41 (1935)) and a method due to Dunham Jackson (this Bulletin, vol. 36 (1930)) lead to the following result: Let C be a rectifiable Jordan curve in the z-plane and let $P_n(z)$ be an arbitrary polynomial of degree n in z. Let $\int_C |P_n(z)|^p |dz| = \epsilon_n$, $p>0$. Then $|P_n(z)| \leq M n^{2/p} \epsilon_n^{1/p}$, z on C, where M is a constant independent of n and z. Let $\{P_n(z)\}$ be the set of polynomials normal and orthogonal on C and let $f(z)$ belong to L^2 on C, and let $f_1(z)=a_0P_0(z)+a_1P_1(z)+\cdots+a_kP_k(z)+\cdots$, $a_k=\int_C f(z)\,\overline{P_k(z)}\,|dz|$. Now by putting $\epsilon_n=1$ and $p=2$ in the above inequality the following result is obtained: If $\sum_{k=0}^{\infty} |a_k| k$ converges, the function $f_1(z)$ is analytic in C, continuous in $\bar{C}$, the closed limited set bounded by C, and $f_1(z)=f(z)$ almost everywhere on C. (Received October 27, 1936.)

425. Dr. C. B. Tompkins (National Research Fellow): *Some integer-valued integral invariants of closed curves and bands.*

A band is a homeomorph in three-space of the portion of a plane between two concentric circles. A contour is a simple closed curve which can not be shrunk on the band to a point. The Gauss integral gives the number of interlinkings of two nonintersecting contours on the band. This integral is split into the sum of two integrals, one depending only on a contour, the other depending on surface-theoretic quantities along this contour (it is a

known integral of G. Y. Rainich). These two integrals are integer-valued. Another integral is introduced, and when added to the Rainich integral it gives an integer which is invariant under locally isotopic deformations. This last integer is the number of local interlinkings of two adjacent contours. When this number is subtracted from the Gauss integral there is left an integer, depending only on the contour of integration, which gives the degree of knottedness of the band or of its contours. (Received November 5, 1936.)

426. Professor T. L. Wade: *Algebraic concomitants by tensor algebra: Reduction.*

In this paper it is shown how certain identities satisfied by the numerical tensors may be used in the reduction of and in the establishment of syzygies on algebraic concomitants constructed by composition of tensors. The fundamental identities are given for the binary domain subjected to the unrestricted linear transformation, and for the ternary domain subjected to the unrestricted linear transformation, and also for the restricted linear transformations (affine and euclidean). While such identities are closely analogous to those of the second fundamental theorem of the Clebsch-Aronhold symbolic method, there are distinctions; for one thing the identities employed here are on real vectors and matrices satisfying the tensor law of transformation as in contrast to the identities of the symbolic method which are on "ideal" vectors, whose elements in general have no particular meaning except when they occur in products involving a specified number of them. (Received November 2, 1936.)

427. Professor T. L. Wade: *An irreducible complete system by tensorial methods of euclidean concomitants for the line and conic.*

By using the method of constructing concomitants by composition of tensors and the tensor identities discussed by the author in a previous paper, an irreducible complete system of euclidean concomitants for the line and the conic is found. This system consists of forty-two concomitants: eight invariants, ten covariants, fifteen contravariants, and nine mixed concomitants. Of these concomitants twelve are algebraically independent; so the forty-two concomitants are connected by thirty syzygies. This set of thirty syzygies is established with the aid of the fundamental identities used in reduction. (Received November 2, 1936.)

428. Professor J. L. Walsh and Professor G. M. Merriman: *Note on the simultaneous orthogonality of harmonic polynomials on several curves.*

Szegö has recently determined (Transactions of this Society, vol. 37 (1935), pp. 196–206) sets of polynomials, $p_n(z)$, in the complex variable z which are simultaneously orthogonal with respect to suitable norm functions on all level curves of a given family. In the present note the authors show that the harmonic polynomials found by separating the $p_n(z)$ into real and pure imaginary parts are also orthogonal on the corresponding level curves, except in Szegö's case II with $\alpha > 2$. (Received May 27, 1936.)

429. Mr. A. L. Whiteman: *On a set of postulates for Boolean algebras by terms of triadic rejection.*

The operation of triadic rejection in Boolean algebra is the operation ( ) given by $(abc)=a'b'+b'c'+c'a'$. It is the object of this paper to present a set of postulates for Boolean algebras in which triadic rejection is taken as the only primitive idea, besides that of class. As a result, all the special Boolean elements are introduced with an elegance not possible in any other set known to the author. Thus the negative of an element is defined in terms of the primitive ideas, and then any two contradictory elements are chosen to represent the zero element and the universe element of the resulting Boolean algebra. (Received October 29, 1936.)

430. Professor Hassler Whitney: *Matrices of integers and combinatorial topology.*

Two types of conditions for the solubility of equations in integers $\sum a_{ij}x_j = y_i$ (or with x_j, y_i elements of an abelian group) are given. The proof is based on a general theorem on the extension of a homomorphism. Cocycles and coboundaries, dual to cycles and boundaries, are characterized in terms of the latter, and vice versa. A theorem relating homology and cohomology groups is given. As an application, a simple basis for the theory of closed and irreducibly closed complexes is given. (Received October 27, 1936.)

431. Dr. J. M. Feld: *Counterpoints and associated cubics.*

Properties of the projective counterpoint (isogonal) transformation and their bearing on the geometry of plane cubic curves are discussed. It is shown that on each line of the plane lies a pair of counterpoints and that the pairs of counterpoints on the lines of a sheaf of lines lie on a cubic curve. Starting with three pairs of counterpoints an infinite number of other pairs can in general be derived from them by straight-edge alone, and these derived pairs as well as the original ones all lie on a cubic curve. (Received October 15, 1936.)

432. Dr. D. T. Sigley: *Orders for which there exist exactly six groups.*

If one prime factor of a number n is congruent to unity with respect to another prime factor of the number n, it is said that n involves a *unity congruence* as defined by G. A. Miller (American Journal of Mathematics, vol. 55 (1933)). The number of abstract groups of a given order is a function of the number of distinct unity congruences in the order. With the aid of the results of the paper cited above, the orders for which there exist precisely six groups are determined; the previous cases have been determined by G. A. Miller (Proceedings of the National Academy of Sciences, vol. 19 (1933), two papers). As a final result an approximation to the asymptotic relation between the number of groups of a given order and the number of unity congruences in that order is given. (Received October 16, 1936.)

433. Mr. P. O. Bell: *A characteristic property of the directrix of Wilczynski.*

The directrix of the first kind which Wilczynski introduced intersects the

tangents to the asymptotic curves of a surface in two covariant points. In this paper a characteristic property of these points is given which associates them with a number of covariant points whose general coordinates are obtained by a sequence of partial differential operations. (Received October 23, 1936.)

434. Professor G. D. Gore: *Transformations of a surface bearing a family of asymptotic curves.*

Two transformations are established for any non-developable surface that bears a family of asymptotic curves and is immersed in a space of n dimensions ($n<3$). The first is an analogue of the transformation of Levy for conjugate nets. The second is an analogue of the fundamental transformation F of Eisenhart. (Received October 24, 1936.)

435. Professor W. G. Warnock: *Triple systems as ruled quadrics.*

The triple of a triple system is considered as a ruled quadric. The number and kinds of intersections of ruled quadrics so formed are studied. (Received October 26, 1936.)

436. Dr. G. M. Ewing: *Sufficient conditions for a non-regular problem in the calculus of variations.*

Usual sufficient conditions for the problem of minimizing $J=\int f(x, y, y')dx$ include the condition (*III'*) that $f_{y'y'}(x, y, y')$ shall not vanish on the minimizing curve. The present paper assumes an arc $E: y=e(x)$ satisfying the necessary conditions of Euler, Weierstrass, and Legendre (I, II, and III). The one parameter family of integrals $L=\int\phi(x, y, y')ds$, $\phi=f(x, y, y')+k^2\cdot[y'-e'(x)]^2$, $k\neq 0$, is then introduced. E is an extremal for L and a further necessary condition for E to minimize J is that it satisfy the Jacobi condition in the strict form (IV') for L. Conditions sufficient for E to furnish an improper strong relative minimum for J are obtained by suitably strengthening these necessary conditions. (Received October 26, 1936.)

437. Dean E. B. Stouffer and Mr. E. W. Emery: *Note on the characterization of Sannia's tetrahedron O.*

In the study of projective differential properties of a curve in ordinary space use is made of a linear homogeneous differential equation of the fourth order. The Halphen canonical form of this differential equation is analytically the simplest canonical form known. Its associated tetrahedron of reference has been located geometrically by Sannia by the use of certain dual properties. In this paper the introduction of dual coordinates is avoided by means of a uniquely determined section of the developable surface generated by the tangents to the curve. (Received October 26, 1936.)

438. Dr. V. P. Jensen and Professor D. L. Holl: *An application of non-analytic functions in plane stress problems.*

In two dimensional plane stress problems, Airy's stress function may be written as a function of the conjugate variables z and $\bar{z}$ so that $F(x, y)$

$=F[(z+\bar{z})/2, (z-\bar{z})/2i]$. It is shown that for the non-analytic function $H=2\partial F/\partial\bar{z}$, the Kasner circle defined by the directional derivative of H is identical with Mohr's circle of stress. Additional properties follow by which one is enabled to discuss the nature of the stresses on curvilinear boundaries as well as the nature of the stresses for the boundary of a new problem obtained by conformal mapping. (Received October 26, 1936.)

439. Dr. M. S. Webster: *Orthogonal polynomials with orthogonal derivatives.*

W. Hahn has proved that orthogonal polynomials which possess orthogonal derivatives are essentially Jacobi, Laguerre, or Hermite polynomials. More recently, Krall gave a new proof of the theorem when the interval of orthogonality is finite. This paper extends this method to the case when the interval is infinite, thus obtaining the Laguerre and Hermite polynomials. (Received October 26, 1936.)

440. Professor W. S. Kimball: *Derivatives of line integrals.*

Formulas for the derivatives of line integrals are developed by a Δ process similar to that employed in deriving the new critical equations for extrema in the calculus of variations. An important advance is made in that attention is not restricted to cases where the integrals are maxima and minima, but expressions are derived for their partial rates of change with respect to x and y at any point on every path of integration admitted to consideration, it being noted how these rates of change, both first and second derivatives, are characterized along the "extremal" arcs. The cases treated are line integrals with fixed integration limits, those with variable upper limit and fixed lower limit, and those with fixed upper and variable lower limit. Derivative formulas are derived not only with respect to variable limits of integration, but with respect to x and y at any point along any path between these limits. Second and higher derivatives of line integrals are also discussed, but attention is restricted to single integration. (Received October 27, 1936.)

441. Professor Arnold Emch: *On symmetric ruled surfaces.*

This paper deals with symmetric ruled and developable surfaces and their properties. In particular it is shown how such surfaces are obtained by their proper analytic representation. Some typical examples are given to illustrate the theory. (Received October 27, 1936.)

442. Professor A. S. Householder: *The dependence of a focal point upon curvature in the calculus of variations.*

Methods of the tensor analysis for a Finsler space of n dimensions as developed by Cartan in a recent monograph are employed to develop a new normal form of the Jacobi equations with $n-1$ instead of n dependent variables. These equations are utilized to generalize results of Bliss for 2-space and of White for 3-space showing that the focal point of a curve transversal to an extremal varies monotonically on this extremal from its intersection with the curve, as the curvature of the curve at this intersection decreases from infinity. Moreover, the focal point approaches the first point conjugate to this inter-

section as the curvature approaches a definite limiting value which is finite or negatively infinite according as the transversal curve is or is not tangent to a certain linear subspace whose dimensionality is less than $n-1$ by the order of the conjugate point. (Received October 27, 1936.)

443. Dr. A. R. Schweitzer: *Projective analogues of descriptive systems of axioms in the foundations of geometry.*

A system of axioms which fall into two categories, (1) primarily without, and (2) with, reference to order, is constructed for projective geometry. In the first category the systems ${}^{n}K_{n}^{*}$ $(n=1, 2, \cdots)$ of axioms which are the analogs of the systems ${}^{n}K_{n}$ of a previous paper (American Journal of Mathematics, vol. 31 (1909), pp. 365–411) are constructed. The former systems are readily obtained from the latter with only two major changes. The systems ${}^{n}K_{n}^{*}$ are sufficient for projective geometry provided axioms are added which imply, under the axioms, the "fundamental theorem" of projective geometry. Also in the first category are constructed projective analogs of descriptive systems due to Veblen (Transactions of this Society, vol. 5 (1904), pp. 343–384) and the author (Transactions of this Society, vol. 10 (1909), pp. 309–314). In the second category fall the systems ${}^{n+1}K_{n}^{(0)}$ which are obtained from the systems ${}^{n}K_{n}$ by "tactical adjunction" (American Journal of Mathematics, vol. 34 (1912), p. 173). For n equal to 1 the system is properly projective. This system is extended to the n-dimensional system ${}^{n+1}K_{2}^{(0)}$. The latter system is then enlarged so as to provide complete sets of axioms for projective geometry in terms of a single undefined relation between ordered triads of points. (Received October 28, 1936.)

444. Professor N. H. McCoy: *Some theorems concerning non-associative rings.*

This paper presents a study of non-associative rings with unit element, special attention being given to the ideals in such rings. If M is a right ideal in the ring R then clearly R and M may be considered as additive groups with the left operator domain R. One of the principal results is an extension of a theorem of Noether and Schmeidler (Mathematische Zeitschrift, vol. 8 (1920), p. 11) concerning the relation between an additive decomposition of R/M and the expression of M as the L.C.M. of a "total teilerfremd" system of ideals. Special attention is also given to the study of two-sided ideals, in which case some of the decompositions are shown to be unique under assumption of the maximum and minimum conditions for two-sided ideals. (Received October 28, 1936.)

445. Professor Gordon Fuller: *Transformation of a system of partial differential equations into tensor form.*

This study represents an extension of an investigation reported before the Society on April 8, 1932 (abstract 38-5-128). A system of equations which has the same general structure as before is considered. The equations have four independent variables, however, instead of three, and are homogeneous in the derivatives. As before, the independent variables are not vector or tensor components; but new variables are introduced which may be considered as the

components of a vector, and new equations are established in terms of this vector which are consequences of the original system. The new system is of the second order, and non-linear. (Received October 29, 1936.)

446. Professor C. C. Camp: *Expansions involving differential equations in which the coefficient of the parameter changes sign.*

In the equation $X'+\lambda a(x)X=0$ the coefficient $a(x)$ is real and integrable such that $\int_{-1}^{1}a(x)dx=A(x)\Big]_{-1}^{1}=2A_1\neq 0$, but otherwise unrestricted Hence it may vanish over a whole subinterval and at any number of other points. Let $f(x)$ be made up of a finite number of pieces, each real, continuous, and possessing a continuous derivative. For the boundary condition $X(-1)=X(1)$ the resulting expansion will converge, but not always to the value of f. Under certain conditions it will converge to f over a whole subinterval. Elsewhere it may converge to the usual mean value plus integral multiples of values of f at other points. A similar situation obtains at boundary points and points x_i where $A(x_i)-A(-1)=k_iA_1$ and k_i is an integer. The theory is extended to the more general case $X'+[\lambda a(x)+b(x)]X=0$, $X(-1)=cX(1)$, as well as to a system in p parameters. (Received October 29, 1936.)

447. Dr. D. G. Bourgin and Dr. R. Duffin: *A method for certain two-point boundary value problems.*

This method is distinct in principle from that of the Green's functions. In the cases to which it applies, a kernel $G(x, n)$ exists such that $MG(x, n)=N(n)\cdot G(x, n)$ where M is the operator adjoint to the original L, and $N(n)$ is a polynomial (though extensions to a linear operator for $N(n)$ are possible). For a suitable domain $\{\phi(x)\}$ defined on an x range properly containing the interval ab, a resolvent kernel $H(n, x)$ and a fixed path l exists such that $\phi(x^+)+\phi(x^-)=2\int_l\int_a^b H(n, x)G(n, t)\phi(t)dtdn=\int_l H(n, x)g(n)dn$. The function $g(n)$ is fixed in terms of the bilinear concomitant $N(n)$, and $\int_a^b G(x, n)f(x)dx$. In conjunction with the boundary relations $U_i(a, b)=\alpha_i$, $i=1, \cdots, n$, the stipulation that $g(n)$ associate an image function vanishing outside of $a\leqq x\leqq b$ determines a solution of the differential system for permissible $f(x)$. For constant coefficient equations $G=e^{-xn}$, $H=e^{xn}$, and the resolvent integral is that of Mellin. The $g(n)$ condition is that it be entire. (Received November 2, 1936.)

448. Professor W. H. Roever: *The deviating force on a rapidly moving aeroplane due to the earth's rotation.*

In order to account for the motion of a body with respect to a frame of reference fixed to the solid part of the earth, two forces are brought into play in addition to those which account for the motion with respect to an inertial frame of reference. These are the centrifugal force and compound centrifugal force. The resultant of the first of these with the earth's gravitational force constitutes weight. In the field of weight the second of these forces causes a moving body to deviate from its direction of motion. It is the purpose of this paper to determine this deviating force for a heavy rapidly moving aeroplane. (Received November 9, 1936.)

449. Dr. Hans Lewy: *A priori limitations for solutions of Monge-Ampère equations.* II.

By means of a regularizing contact transformation that preserves the Monge-Ampère type of the equation a certain convergence theorem for solutions of analytic elliptic Monge-Ampère equations is reduced to the result of first part of this paper (Translations of this Society, vol. 35 (1935), pp. 417–434). Additional conditions granting the analyticity of the limit function are established. (Received October 20, 1936.)

450. Dr. M. R. Hestenes: *A direct sufficiency proof for the problem of Bolza in the calculus of variations.*

In the present paper a direct sufficiency proof is given for the problem of Bolza which is independent of assumptions of normality. Heretofore sufficiency proofs for the problem of Bolza independent of assumptions of normality have been indirect in the sense that they depended on a transformation of the problem into an equivalent one of somewhat different form. No transformation is needed in the present paper. The relation between the present method and the earlier ones is also discussed. (Received October 26, 1936.)

451. Professor A. A. Shaw: *On Measures and Weights by Epiphanius.* II.

For the first part of this paper see abstract 42-5-247, and National Mathematics Magazine, vol. 11 (1926), pp. 3–7. This second part deals with a new analysis of the character of Epiphanius based upon a study of his extant letters to Bishop John of Jerusalem and to Saint Jerome; and of the works of his contemporaries (Saint Jerome, Socrates Scholasticus, Salaminus Hermias Sozomen, Theodoret, and others) as well as those of modern critics such as R. A. Lipsius, Phillip Schaff, and Bardenhewer. (Received October 26, 1936.)

452. Professor P. N. Daus: *The ternary cubic representing norm and its hessian.*

If $N(x, y, z)$ represents the norm of a cubic algebraic integer, and H is the hessian of $N=0$, it is shown that $H=2DN$, where D is the discriminant of the cubic irrationalities used as a basis. (Received October 28, 1936.)

453. Professor W. M. Whyburn: *Note on functional equations of a cyclic type.*

The functions determined by equations of the form $L(y_i)=\sum_{j=1}^{n}A_j(x)y_{\alpha i+\beta i+\gamma}$, where L is a linear operator and α, β, γ, n, are integers, constitute a highly useful group of functions. A former paper (this Bulletin, vol. 36, pp. 863–868) treated the case of these equations where $\alpha=1$. The present paper studies cases that arise when $\alpha<1$. The special case $\alpha=n-1$ is treated in detail and is shown to be equivalent to a pair of equations of the type $L(u)=B(x)\bar{u}$, $L(\bar{u})=\overline{B}(x)u$, where u and B are complex functions of the real variable x and $\bar{u}$ is the conjugate of u. Known results for second order, self-adjoint differential equations are used in a study of cases that arise when $L(y_i)=dy_i/dx$. (Received October 28, 1936.)

454. Professor Morgan Ward: *The ranks of apparition of primes in a linear divisibility sequence.*

The sequence of rational integers $(u): u_0, u_1, u_2, \cdots$, is said to form a linear divisibility sequence if it satisfies a linear recursion relation with integral coefficients and if u_n divides u_m whenever n divides m. We here attack the problem of determining a priori all terms of (u) divisible by a pre-assigned modulus. We give a complete solution of this problem by the methods of ideal theory for the important case of prime moduli dividing neither the discrminant of the characteristic polynomial of degree k associated with the recurrence nor the persymmetric determinant $\Delta(u)$ of order k built from the first $2k-1$ terms of (u). The problem is essentially deeper than the previously treated problems of determining the period and numeric of any linear sequence (Transactions of this Society, vol. 35 (1933): pp. 600–628) in that the nature of the group of the characteristic polynomial must be taken into account. (Received October 30, 1936.)

455. Professor Glenn James: *A higher upper limit to the parameters in Fermat's equation,* $x_1^n + x_2^n = x_3^n$.

This paper treats the so-called first case of Fermat's Last Theorem, in which the parameters x_i are not divisible by n. It has been previously proved that this case of the theorem holds for all $x_1 \leqq n(cn+1)^n$ where c is even and $x_1 < x_2 < x_3$. The present paper proves that c contains n as a factor. Since the theorem has been proved for $n = 14000$, it is now seen to be true for all values of n provided $x_1 \leqq 14000\ (312{,}000{,}000)^{14000}$. (Received October 30, 1936.)

456. Professor A. D. Michal and Mr. D. H. Hyers: *Differential invariants in an abstract differential geometry.*

In this paper the authors continue their study of abstract normal coordinates in a general differential geometry (see abstract 42-9-348). An existence theorem is proved for a certain functional equation in an abstract euclidean space. For the definition of such a space see A. D. Michal, I. E. Highberg, and A. E. Taylor, *Abstract euclidean spaces with independently postulated analytical and geometrical metrics* (accepted for publication by Tonelli's Annali di Pisa). The theorem is used to show that normal coordinate methods may be applied to the absolute calculus of forms in *covariant* as well as in contravariant vector fields. There results a replacement theorem for abstract differential invariants. (Received October 30, 1936.)

457. Mr. E. W. Paxson: *Existence theorems for ordinary differential equations in topological spaces.* Preliminary report.

In this note the author continues the discussion of differential equations in a non-metric linear topological space, L (see abstracts 42-9-355, 42-5-137). The problem of Cauchy is investigated for the equation $y'(\mu) = f(y, \mu)$ where $f(y, \mu)$ is a continuous function on a region in the topological space L, and on a real interval, to the space L. (Received October 31, 1936.)

458. Dr. A. E. Taylor: *Analytic functionals defined in Hilbert space.* Preliminary report.

In this paper the abstract theory of analytic functions previously announced (abstracts 41-11-424, 42-3-147, and 42-9-369) is applied to the study of analytic complex functionals $f(z)=f(z_1, z_2, \cdots)$ defined on open subsets of the complex Hilbert space H_0. The explicit form of the power series expansion of $f(z)$ about a regular point is obtained. The homogeneous polynomials which form the series are forms of the nth degree in an infinite number of variables, bounded in the Hilbert sense, with coefficients given in terms of the partial derivatives of $f(z_1, z_2, \cdots)$, evaluated at the point in question. By an example of the above sort it is demonstrated that the author's theory is more general than that of R. S. Martin (see, for instance, Comptes Rendus, vol. 197 (1933), p. 735), in that, although the two definitions of analyticity are locally equivalent, the radius of analyticity about a given point, as defined by Martin, is in certain cases smaller than as defined by the author, whereas all fundamental properties of the function obtain in the larger region. (Received October 31, 1936.)

459. Professor Max Zorn: *A remark on real division algebras.*

The author investigates the consequences of a generalized associative law in division algebras over the field of all real numbers. (Received October 31, 1936.)

460. Professor E. T. Bell: *Certain ternary cubic arithmetical forms.*

By an obvious oversight, G. B. Mathews (*Proceedings of the London Mathematical Society*, vol. 21 (1891), pp. 280–287) gave an incomplete statement of a special case of Dirichlet's theorems on representations by norms of algebraic integers. Rectifying the oversight, we find forms for which the number of representations of any integer is finite. For example, 9 p, where p is a prime $6n+1$, has precisely 18 representations in $x^3+y^3+z^3-3xyz$. (Received November 2, 1936.)

461. Mr. Victor Elconin: *Composite operators.*

Let S be a set of sets. For $\sigma \subset S$ let $\mu\sigma$ be the union of the elements of σ. For any *operator* T in S, $T\sigma$ is a function on a subset of S to S. The *range* ρT and the domain δT of T are those of $T\sigma$. T is *composite* if (i) $\sigma \subset \mu\delta T$ implies $\sigma \epsilon \delta T$, and (2) $\sigma \epsilon \delta T$ implies $T\sigma$ is the union of the set of all Tf for $f \epsilon \sigma$. Only composite operators are considered here. T is *simple* if $f \epsilon \mu\delta T$ implies $Tf \epsilon \mu\rho T$. Definitions of T^{-1}, if T is simple, and of TU are given. T is of *index* n if for any $s_1, s_2, \cdots, s_r$ in μS there is a unique $f \epsilon \delta T$ such that $f=s_1$, $Tf=s_2, \cdots, T^{n-1}f=s_n$. Linear operators, defined when $\mu\delta$ is a linear space, are studied in detail, and the results obtained are illustrated by special cases, including that of the differentiator $T=\sum_0^n a_k D^k$ in S where $\mu\delta$ is a function space. The entire theory of T^{-1} is obtained without recourse to Fuchsian existence or uniqueness theorems. (Received November 2, 1936.)

462. Dr. Hans Lewy: *Generalized integrals and differential equations.*

The notion of an integral $\int a(x_1)(x))db(x_1 f(x))$, in which a and b are continuously differentiable functions of *two* variables and $f(x)$ is a function of Baire's first class, is introduced and certain continuity properties of it are proved. Similarly "solutions" of differential equations are defined and determined in the case that the equation involves the derivative of a function $f(x)$ of which no more is known than that it belongs to Baire's first class. The results obtained imply new types of à priori limitations for solutions of differential equations, and new existence theorems in the large. (Received October 20, 1936.)

463. Dr. Hans Lewy: *On differential geometry in the large.*

The results of the preceding paper are applied to Minkowski's and Weyl's problems: to determine closed convex surfaces whose curvature is positive and given as function of the direction of the normal (Minkowski) or whose metric is given (embedment problem in the large). In both cases existence of analytic surfaces solving the proposed problem is shown for the case of analytic data satisfying the necessary equalities. (Received October 20, 1936.)

464. Mr. Alvin Sugar: *Waring's problem for almost nth powers.* Preliminary report.

We shall call a member of the class $K_{nt}=[0, 1, x^n+t]$, where n is a positive integer and x and t are non-negative integers, an almost nth power of order t. The purpose of this paper is to evaluate $g(K_{nt})$, which is defined as the least number of almost nth powers of order t required to represent every positive integer. (Received October 20, 1936.)

465. Professor Einar Hille: *On bilinear formulas and Laplace transforms.*

This note contains a study of the application of bilinear series for the kernel $e^{-xu}\sim\sum_0^\infty \phi_n(u)\ \psi_n(z)$, $0<u<\infty$, $0<R(z)$, to the transformation of Laplace. Here $\{\phi_n(u)\}$ is a complete orthonormal system in $L_2(0, \infty)$. If $F(u)\sim\sum_0^\infty a_n \cdot \phi_n(u)$ and $f(z)=\int_0^\infty e^{-zu}F(u)\,du$, then $f(z)\sim\sum_0^\infty a_n\psi_n(z)$. If $F(u)\in L_2(0, \infty)$, the first series is the Fourier orthogonal series of $F(u)$ and the second one converges absolutely to a function $f(z)\in H_2(0)$. The relationship between the two series persists in more general cases, however, if $\{\phi_n(u)\}$ is suitably chosen, and can be used to obtain inversion formulas for the Laplace and Laplace-Stieltjes integrals. Special cases of such formulas have been found by F. Tricomi and D. V. Widder, using Laguerre polynomials, and by F. Hausdorff, using Legendre polynomials. (Received October 28, 1936.)

466. Profesor R. D. James: *On a problem in additive number theory.* Preliminary report.

A short time ago the author was asked by Professor D. N. Lehmer if there were any results on the representation of an integer as a sum of integers all of whose prime factors were of a certain linear form. The purpose of this paper is to give results of this type in two directions. First of all, it follows immedi-

ately from Schmielmann's general theorems (Iswestija Donskowo Polytechnitscheskowo Instituta (Nowotscherkask), vol. 14 (1930)) that every integer is a sum of a finite number of integers, all of those prime factors are of the form $ak+b$, where a and b are any relatively prime integers. Secondly, by considering certain quadratic forms, results can be obtained of which the following is an example. Every integer is a sum of at most 7 integers which are either 1 or have all their prime factors of the form $8k+1$. The number 7 cannot be replaced by any smaller number. (Received October 26, 1936).

467. Professor P. H. Daus: *A geometric interpretation of the convergents for a ternary continued fraction.* Preliminary report.

If θ is the root of an irreducible cubic equation of negative discriminant, we consider the point $P(x_n, y_n, z_n)$, where x_n, y_n, z_n is the nth convergent set of the ternary continued fraction determined by θ, and the family of syzygetic cubic curves $N(x, y, z)=t_nxyz$, where t_n is determined by the nth convergent set. Each curve is a two-circuited cubic, containing one and only one convergent point. The ovals form a non-intersecting nested set converging to the cubic irrationalities in question. This property is being made the basis of a study of the choice of the partial quotients in the ternary continued fraction expansion. (Received October 28, 1936.)

468. Professor Arnold Emch: *On certain configurations of points in space and linear systems of surfaces with these as base points.*

This paper deals with the configuration of points G_{27} of points W defined by $W(w^\alpha, w^\beta, w^\gamma, 1)$, $\alpha, \beta, \gamma \equiv 0, 1, 2, \pmod 3$, A_1 (1000), A_2 (0100), A_3 (0010), A_4 (0001); and the G_{36} of points V, which outside of the A's are the vertices of the 4 flex triangles of the syzygetic pencil of cubics of each face. Connected with these as base-points are linear systems of surfaces of which some typical examples are investigated. (Received October 29, 1936.)

469. Mr. D. H. Hyers: *Lebesgue integrals for functions out of a quasi-convex linear topological space.* Preliminary report.

The definition of Lebesgue integrals adopted by Garrett Birkhoff for functions with values in a Banach space (Transactions of this Society, vol. 38 (1935), p. 357) is extended to the case where the Banach space is replaced by a linear topological space. It is assumed that the space is complete and that every neighborhood satisfies a certain convexity condition. (Received October 31, 1936.)

470. Professor A. D. Michal: *Differential geometry of an abstract topological group with Banach coordinates.* Preliminary report.

This paper deals with a new chapter in the author's abstract differential geometries with coordinates in a Banach space. The geometry of Lie continuous groups studied by Cartan, Eisenhart, Schouten, and many others during

the last decade is brought under this more general theory. (Received October 31, 1936.)

471. Mr. D. H. Hyers: *Gateaux differential equations in Banach spaces.*

A. D. Michal and V. Elconin have established an existence theorem for the total differential equation $y(x;\xi)=F(x,y(x),\xi)$ in a Banach space, where F is linear in ξ and the left member is the Fréchet differential of the unknown function $y(x)$ (see *Completely integrable differential equations in abstract spaces*, Acta Mathematica (in press) or, Proceedings of the National Academy of Sciences, vol. 21 (1935), p. 534). The author proves an existence theorem for this equation in which F is merely additive and homogeneous but not necessarily continuous in ξ, and where the left member is interpreted as a Gateaux differential. (Received October 31, 1936.)

472. Mr. Victor Elconin: *The Wronskian of the functions* $x^{k-1}e^{\alpha_j x}$.

It is shown by three distinct methods that the Wronskian of the $n_1+n_2+\cdots+n_r$ functions $e^{\alpha_1 x}$, $xe^{\alpha_1 x}$, $\cdots$, $x^{n_1-1}e^{\alpha_1 x}$, $e^{\alpha_2 x}$, $\cdots$, $x^{n_r-1}e^{\alpha_r x}$, where the α_k are complex numbers, is

$$\prod_1^k (n_j-1)!!\prod_{r\geqq j>k\geqq 1}(\alpha_j-\alpha_k)^{n_j-n_k}e^{(n_1\alpha_1+n_2\alpha_2+\cdots+n_r\alpha_r)x}$$

where $m!!=1!2!\cdots m!$. (Received November 2, 1936.)

473. Mr. Victor Elconin: *Linear paraphrasing.*

E. T. Bell's fundamental theorem on arithmetical paraphrasing can be generalized and abstracted to describe all similar linear substitutions. Let S be a finite subset of a linear space, and σ any other finite set. S is *represented by* f_s *on* σ if the f_s for $s\epsilon\sigma$ are all the elements of S. $\rho\subset\sigma$ belongs to $R\subset S$ if R is represented by f_s on ρ. R is *irreducible* if $\sum_r^\rho \alpha_r f_r=0$ implies $\alpha_r=0$, *reducible* otherwise; the α_r, as in the following, are complex numbers, and the summation is for all $r\epsilon\rho$. R is *maximal* if it is irreducible and $R\subset U\subset S$, $R\neq U$, implies U is reducible. The basic theorem follows: *Suppose that* S, T *are finite subsets of linear spaces represented by* f_s, g_s *on* σ, *and that* ρ *belongs to a maximal subset of* S. *If* $f_s=\sum_r^\rho \alpha_{sr}f_r$ *implies* $g_s=\sum_r^\rho \alpha_{sr}b_r$, *then* $\sum_s^\sigma \alpha_s f_s=0$ *implies* $\sum_s^\sigma \alpha_s g_s=0$. A series of increasingly specialized sets S, T are considered and the characteristic property of ρ is determined in each case, for the last of which Bell's theorem (see Transactions of this Society, vol. 22 (1921), p. 5), generalized to arbitrary complex coefficients, is a corollary of the theorem above. (Received November 2, 1936.)

474. Mr. Victor Elconin: *Euler's integrability condition.*

Sarrus, De Morgan, Raabe, Joachimstal, and several others have given proofs none of which involves variational methods of Euler's theorem:

$F(x, y, dy/dx, \cdots, d^ny/dx^n)$ is an exact derivative if and only if $\partial F/\partial y - d/dx$ and $\partial F/\partial y_1 + \cdots + (-)^n d^n/dx^n$ and $\partial F/\partial y_n = 0$, where $y_k = d^ky/dx^k$. The inductive proof here presented appears to be the shortest elementary proof yet devised. (Received November 2, 1936.)

ERRATUM

On page 497 of this volume, abstract 42-7-311 (by Professor C. C. Camp), in the statement of the lemma, replace the words "otherwise zero" by "and $h-\theta$ when $h=h'+\theta$, $0<\theta<1$, where h' is an integer."

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